MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS, ABSTRACTS OF PAPERS, AND

REPORTS OF THE PROCEEDINGS

OF THE SOCIETY

FROM NOVEMBER 1902 *TO NOVEMBER* 1903.

(WITH TWO APPENDICES)

VOL. LXIII.

LONDON:
ROYAL ASTRONOMICAL SOCIETY
BURLINGTON HOUSE, W.
1903.

PRINTED BY
SPOTTISWOODE AND CO. LTD., NEW-STREET SQUARE
LONDON

BIBL.
LONDIN.
UNIV.

INDEX.

PAGE

Albrecht, Th., extract from a letter from, in reference to the establishment of a southern belt of latitude stations 394
Aldis, W. S., note on the use of his tables of the function $\frac{1}{2}(\theta + \cos\theta)$, H. C. Plummer 147
Almucantar, the Durham, transits observed with the, 1901–2, R. A. Sampson 338
Annual General Meeting, 1903, February 13, report of the..... 177
Antoniadi, E. M., observation of the partial eclipse of the Moon, 1903 April 11..... 400
Assets and property of the Society, 1903 183
Associates proposed 393
Astrographic chart, note on the reproduction and publication of the photographs taken for the, at the Royal Observatory, Greenwich 132
——————— and catalogue, Council note on 269
Aubertin, J. J., obituary notice of..... 196
Auditors, report of the..... 182

Bellamy, F. A., note relating to the preservation of negatives 71
——————— note on a new star in the constellation of *Gemini* 326
——————— positions of 166 stars around Nova *Geminorum*; with a discussion of systematic differences between two exposures on the same plate 512
——————— on the position of *X Geminorum*..... 526
——————— and H. H. Turner, announcements respecting a paper on the possible existence of two independent stellar systems ...119, 326
Bowyer, W., on the orbit of Σ 2525 334
Brown, E. W., on the verification of the Newtonian law 396
Burnaby, Rev. S. B., obituary notice of..... 196
Burnham, S. W., note on the double star 31 *Leonis*..... 419
——————— new companion to Σ 1594 420
Bye-law 23, alteration of... 177

Cambridge Observatory, report of the proceedings of 220
Candidates proposed 1, 55, 178, 293, 325, 393, 423
Cape of Good Hope, Royal Observatory, double star results, 1902, R. T. A. Innes 76
——————————————— report of the proceedings of ... 212
Chandler, S. C., letter from, proposing the establishment of a southern belt of latitude stations 294
——————————— Council resolution respecting the above ... 296
——————————— extract from a letter from Dr. Th. Albrecht in reference to the above..... 394

PAGE

Chree, C., preliminary note on the relationships between sun-spots and terrestrial magnetism ..... Appendix [2]
Clerke, Miss A. M., elected an Honorary Member 393
Comets:—
b 1902 (Perrine), note on photographs of, taken at the Royal Observatory, Greenwich 34
b 1902 (Perrine), stereoscopic pictures of, Max Wolf 35
b 1902 (Perrine), observed at the Liverpool Observatory, W. E. Plummer 88
b 1902 (Perrine), photographic observations at the Royal Observatory, Greenwich 313
d 1902 (Giacobini), note on photographs taken at the Royal Observatory, Greenwich 134, 315
a 1903 (Giacobini), note on photographs of, taken at the Royal Observatory, Greenwich 315
Council note on the discovery of, 1902 241
Composition fees, alteration of bye-law relating to 177
Cooke, W. E., on a new and accurate method of determining time, latitude, and azimuth with a theodolite 156
Cooper, Col. E. H., obituary notice of 197
Cornu, M. A., obituary notice of 201
Cortie, Rev. A. L., the spectra of sun-spots in the region B—D 468
Crommelin, A. C. D., ephemeris for physical observations of the Moon for 1903 43
——————— ephemeris for physical observations of *Jupiter*, 1903–4 96
Crossley, Edward, report of his observatory 228

Denning, W. F., rotation period of the markings on *Jupiter* 331
——————— recent observations of *Mars* and *Jupiter* 499
Duke, P. F., death of 196
Dunsink Observatory, report of the proceedings of 222
Durham almucantar, transits observed with the, 1901–2, R. A. Sampson 338
Durham Observatory, report of the proceedings of 223
Dyson, F. W., note on plate constants 134

Edinburgh, Royal Observatory, report of the proceedings of 218
Errata 54, 324
Espin, Rev. T. E., new double stars detected with the 17½-inch reflector during the year 1902 172
——————— report of his observatory 228
Evans, J. E., and E. W. Maunder, experiments as to the actuality of the "Canals" observed on *Mars* 488

Faye, Hervé, obituary notice of 204
Fellows elected 1, 55, 119, 178, 325, 393, 423
Folie, F., expressions correctes de l'heure et des coordonnées des étoiles dans le système de l'axe instantané 444

Gill, Sir David, on his suggestion that the brighter fixed stars are as a whole rotating with respect to the fainter stars as a whole, H. H. Turner 56
Glasgow Observatory, report of the proceedings of 223
Gold Medal, the, presented to Professor Hermann Struve for his work on the satellites of *Saturn* 177, 194, 289
Gore, J. E., on the Sun's stellar magnitude and the parallax of binary stars 164

PAGE

Graphical method of applying to photographic measures the terms of the second order in the differential refraction, A. R. Hinks ... 138
Gravity, Council note on Mr. Whittaker's investigations 258
——— examination of Mr. Whittaker's undulatory explanation of, from the physical standpoint, G. J. Stoney 424
Greenwich, Royal Observatory, note on photographs of comet *b* 1902 (Perrine) 34
——— note on a comparison of Groombridge's catalogue (1810) with the Greenwich second ten-year catalogue (1890), with reference to the question of an apparent rotation of the brighter fixed stars as a whole with respect to the fainter stars 36
——— statistics of stars in a zone of 5° from +65° to +70° decl., counted on photographs for the Astrographic Chart and Catalogue 120
——— note on the reproduction and publication of the photographs for the Astrographic Chart..... 132
——— note on photographs of comet *d* 1902 (Giacobini) obtained with the 30-inch reflector 134, 315
——— observations of occultations of stars by the Moon made in the year 1902 174
——— report of the proceedings of..... 207
——— observations of comet *b* 1902 (Perrine) from photographs..... 313
——— note on photographs of comet *a* 1903 (Giacobini) taken with the 30-inch reflector 315
——— eclipse of the Moon of 1903 April 11 ... 402
——— results of micrometric measures of double stars made with the 28-inch refractor in the year 1902 403
——— mean daily areas of sun-spots for each degree of solar latitude for each year from 1874 to 1902, as measured on photographs..... 452
——— areas of faculæ and sun-spots compared with diurnal ranges of magnetic declination, horizontal force and vertical force, as observed in the years 1873 to 1902 462
——— mean areas and heliographic latitudes of Sun-spots in the year 1902, deduced from photographs taken at Greenwich, at Dehra Dûn (India), and in Mauritius 464
——— observations of the satellite of *Neptune*, from photographs taken between 1902 November 12 and 1903 April 27..... 503
Griffin, E. J., obituary notice of 198
Groombridge's Catalogue (1810), comparison of, with the Greenwich second ten-year catalogue (1890), Royal Observatory, Greenwich 36

Hardcastle, J. A., note on binding together réseaux and plates 72
Herman, R. A., a derivation of Hill's equation by a direct substitution 571
Herschel, W., observed nebulous regions compared with photographs, Isaac Roberts 26
Hilger, Otto, obituary notice of 199
Hill's equation, a derivation of, by a direct substitution, R. A. Herman 571
Hinks, A. R., a graphical method of applying to photographic measures the terms of the second order in the differential refraction..... 138
Honorary members elected 393
Huggins, Lady, elected an Honorary Member 393
Huggins, Sir W., report of his observatory 229

Innes, R. T. A., Cape double star results, 1902 76

PAGE

Oxford, Radcliffe Observatory, occultations observed during the lunar eclipse of 1903 April 11 398
——————————— further observations of the new star in *Auriga*, with the mean magnitude for the years 1892–1903 ... 508
——————————— further observations of the new star in *Perseus* 509
——————————— observations of the new star in *Gemini*... 510
Oxford, University Observatory, report of the proceedings of..... 226

Papers read before the Society, 1902–3, list of..... 271
Peirce's criterion for the rejection of doubtful observations, note on the use of, S. A. Saunder 432
Perth Observatory, West Australia, report of the proceedings of 235
Photographic measures: Sur la précision des mesures photographiques: réponse à M. Plummer, M. Lœwy 2
——————————— on the accuracy of; third note; reply to M. Lœwy, H. C. Plummer 14
——————————— graphical method of applying terms of the second order to, A. R. Hinks 138
Photographs, celestial, list of reproductions of 186
——————————— by G. W. Ritchey, recently presented to the Society, note on..... 395
Plate constants, note on, F. W. Dyson 134
Plates:—
Photographs of nebulosities in *Orion* and *Cygnus*, I. Roberts 32
Photograph of comet *b* 1902 (Perrine), R. Observatory, Greenwich 34
Micrometer for measuring star positions, H. C. Russell..... 40
Diagram of number of stars per square degree shown on plates, R. Observatory, Greenwich 128
Three diagrams, graphical method, A. R. Hinks 142
Nebulosities in *Orion*, Max Wolf 303
Light curve of *UY Cygni*, A. S. Williams..... 308
Region of Nova *Geminorum*, Max Wolf 330
Orbit of Σ 2525, W. Bowyer 334
Mean daily areas of sun-spots, 1874–1902, Royal Observatory, Greenwich 452
Areas of faculæ and sun-spots and ranges of magnetic declination, &c., Royal Observatory, Greenwich 462
Curves of sun-spots, prominences, and Corona, W. J. S. Lockyer ... 484
Experiments as to the actuality of the Canals of *Mars*, J. E. Evans and E. W. Maunder..... 494
Nebulosities in *Auriga*, Max Wolf..... 506
Light curve of *RR Centauri*, A. W. Roberts..... 540
System of *RR Centauri*, A. W. Roberts 548
Solar prominences and terrestrial magnetism, Sir N. and W. J. S. Lockyer .....Appendix [6], [8]
Solar prominence and spot circulation, Sir Norman and W. J. S. Lockyer .....Appendix [14]
Plummer, H. C., réponse à deux notes de, M. Lœwy 2
——————————— on the accuracy of photographic measures. Third note: reply to M. Lœwy 14
——————————— on the images formed by a parabolic mirror. Second paper: influence on the measurement and reduction of a photograph 16
——————————— on Jacobi's method of facilitating the numerical solution of equations arising in the theory of secular perturbations 90
——————————— note on the use of Mr. Aldis's Tables of the function $\frac{1}{2}(\theta + \cos\theta)$ in determining the elements of an orbit 147
——————————— on oscillating satellites 436

PAGE

Plummer, W. E., cometary observations at the Liverpool Observatory [1902] 88
Presents announced 2, 55, 119, 293, 325, 394, 423
Progress and present state of the Society 179
Publications, stock of 184
———— of the Society 195

Rambaut, A. A., observations of stars occulted by the Moon during the eclipse of 1903 April 11 at the Radcliffe Observatory, Oxford 398
Report of the Council to the 83rd Annual General Meeting 179
Réseaux and plates, note on binding together, J. A. Hardcastle 72
Ritchey, G. W., note on celestial photographs made by, at the Yerkes Observatory, and recently presented to the Society 395
Roberts, A. W., report of his observatory 237
———— on the relation existing between the light changes and the orbital elements of a close binary system, with special reference to the figure and density of the variable star *RR Centauri* 527
Roberts, Isaac, William Herschel's observed nebulous regions, 52 in number, compared with Isaac Roberts's photographs of the same regions taken simultaneously with the 20-inch reflector and the 5-inch Cooke lens 26
———— report of his observatory 230
———— on the nebula *h* 2302 N.G.C. 7822 *Cassiopeiæ*; the region surrounding H II. 457 N.G.C. 1665 *Eridani*, with ten new nebulæ; and H III. 558 N.G.C. 7492 *Aquarii* 301
Robinson, W. H., photographic and visual magnitudes of *α Orionis* 74
Rousdon Observatory (the late Sir C. E. Peek's), report of the proceedings of 229
Rugby, Temple Observatory, report of the proceedings of 227
Russell, H. C., another form of micrometer for measuring star positions 39

Sampson, R. A., transits observed with the Durham almucantar between 1901 November 23 and 1902 December 15 338
Satellites, oscillating, H. C. Plummer 436
Saunder, S. A., report of his observatory 232
———— note on the use of Peirce's criterion for the rejection of doubtful observations 432
Snow, Robert, catalogue of right ascensions of 76 stars 552
———— 125 stars 556
Solar activity in 1902, Council note on 249
Solar motion in space, Council note on determinations of the 252
Standard scale for telescopic observations, Lowell 40, 337
Star Catalogues:—
Comparison of Groombridge's catalogue (1810) with the Greenwich second ten-year catalogue (1890), Royal Observatory, Greenwich 36
Newcomb's fundamental catalogue; notes and errata, W. G. Thackeray 38
Council note on 268
Snow, Robert, catalogue of right ascensions of 76 stars 552
———— 125 stars 556
Star magnitudes: Note on the photographic and visual magnitudes of *α Orionis*, W. H. Robinson 74
Stars, brighter, apparent rotation of, with respect to the fainter stars: comparison of Groombridge's catalogue (1810) with the Greenwich second ten-year catalogue (1890), Royal Observatory, Greenwich 36

PAGE

Stars, brighter, apparent rotation of, with respect to the fainter stars: on Sir David Gill's suggestion, H. H. Turner 56

Stars, double:—
Cape double star results, 1902, R. T. A. Innes..... 76
On the parallax of binary stars, J. E. Gore 164
New double stars detected with the 17¼-inch reflector in 1902, Rev. T. E. Espin 172
Council note on 260
Σ 2525, orbit of, W. Bowyer 334
Results of micrometric measures with the 28-inch refractor at the Royal Observatory, Greenwich, 1902..... 403
31 *Leonis*, note on, S. W. Burnham 419
Σ 1594, new companion to, S. W. Burnham 420
On the relation between the light changes and orbital elements of a close binary system, A. W. Roberts 527

Stars, new:—
Nova *Aurigæ*, observed at the Radcliffe Observatory, Oxford..... 508
Nova *Persei*, Council note on 258
Nova *Persei*, observed at the Radcliffe Observatory, Oxford 509
Nova *Geminorum*, note on, F. A. Bellamy..... 326
——————— the place of, Max Wolf 330
——————— observed at the Radcliffe Observatory, Oxford ... 510
——————— positions of 166 stars near, with a discussion of systematic differences between two exposures on the same plate, F. A. Bellamy..... 512
——————— on the possible identity of, with a small star photographed before the outburst, H. H. Turner 565

Stars, spectra of:—
Council note on stellar spectroscopy in 1902 264

Stars, statistics of, in a zone of 5° from +65° to +70°, counted on photographs for the Astrographic Chart and Catalogue at the Royal Observatory, Greenwich 120

Stars, variable:—
Council note on..... 262
On the period and light curve of (7514) *UY Cygni*, A. S. Williams 304
X Geminorum, position of, F. A. Bellamy..... 526
RR Centauri, light curve, orbital elements, &c., A. W. Roberts ... 536

Stars, velocity of, in the line of sight: selected stars; Cambridge Observatory, I. 1902, H. F. Newall..... 296

Stoney, G. Johnstone, examination of Mr. Whittaker's "undulatory explanation of gravity" from the physical standpoint 424

Stonyhurst College Observatory, report of the proceedings of..... 227

Struve, Hermann, the Gold Medal presented to him for his work on the satellites of *Saturn* 177, 194, 281

Sun, prominences, 1902, Council note on 251
——————— and corona, probable relationship between, W. J. S. Lockyer 481
——————— of the, and terrestrial magnetism, the relation between, Sir J. N. Lockyer and W. J. S. Lockyer .....Appendix [6]
——— prominence and spot circulation, Sir Norman and W. J. S. Lockyer Appendix [14]

Sun-spots and faculæ, &c.:—
Council note on solar activity in 1902 249
Mean daily area of sun-spots for each degree of solar latitude for each year from 1874 to 1902 as measured on photographs at the Royal Observatory, Greenwich 452
Areas of faculæ and sun-spots compared with diurnal ranges of magnetic declination, horizontal force and vertical force, as observed at the Royal Observatory, Greenwich, in the years 1873 to 1902..... 462

PAGE

Mean areas and heliographic latitudes of sun-spots, 1902, deduced from photographs taken at Greenwich, at Dehra Dûn (India), and in Mauritius 464
The spectra of sun-spots in the region B—D, Rev. A. L. Cortie ... 468
Preliminary note on the relationships between sun-spots and terrestrial magnetism, C. Chree .....Appendix [2]
Solar prominence and spot circulation, 1872–1901, Sir Norman and W. J. S. Lockyer .....Appendix [14]
Sun, stellar magnitudes of the, J. E. Gore 164
Sydney and Melbourne Observatories, joint report on measurement of astrographic plates 235

Tables of $\frac{1}{2}(\theta + \cos\theta)$, by W. S. Aldis, note on the use of, in determining the elements of an orbit, H. C. Plummer 147
Tebbutt, John, report of his observatory 237
Telescopes, whose relative or absolute focal length shall be invariable at all temperatures, on the construction of, F. L. O. Wadsworth 573
Thackeray, W. G., Newcomb's fundamental catalogue. Notes and errata 38
Theodolite, method of determining time, latitude, and azimuth with a, W. E. Cooke 156
Thome, J. M., the national Argentine Observatory 549
Treasurer's account, 1902 180
Trust Funds 182
Turner, H. H., on the suggestion made by Sir David Gill that the brighter fixed stars are as a whole rotating with respect to the fainter stars as a whole 56
——————— address on presenting the Gold Medal to Professor Hermann Struve for his work on the satellites of *Saturn* 281
——————— on the possible identity of Nova *Geminorum* with a small star photographed before the outburst 565
——————— and F. A. Bellamy, announcements respecting a paper on the possible existence of two independent stellar systems 119, 326

Wadsworth, F. L. O., on the construction of telescopes whose relative or absolute focal length shall be invariable at all temperatures... 573
Whittaker, E. T., Council note on his investigations 258
——————— examination of his "undulatory explanation of gravity" from the physical standpoint, G. J. Stoney 424
Williams, A. Stanley, on the period and light curve of (7514) *UY Cygni* 304
Wilson, W. E., report of his observatory 233
Wiltshire, Rev. T., obituary notice of 200
Wolf, Max, stereoscopic pictures of Comet Perrine 35
——————— on three of Sir William Herschel's observed nebulous regions in *Orion* 303
——————— the place of Nova *Geminorum* 330
——————— the great nebula in *Auriga* 506

Yerkes Observatory, note on celestial photographs made at the, by G. W. Ritchey 395

PRINTED BY
SPOTTISWOODE AND CO. LTD., NEW-STREET SQUARE
LONDON

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIII. NOVEMBER 14, 1902. No. 1

Dr. J. W. L. GLAISHER, M.A., F.R.S., PRESIDENT, in the Chair.

Francis Kennedy McClean, Rusthall House, Tunbridge Wells ; and
Rev. Edwin Albert Phillips, B.A., Lecturer on Mathematics, Training College, Exeter,

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Henry Bourget, D. ès Sc., Maître de Conférences à l'Université de Toulouse et Astronome-adjoint à l'Observatoire de Toulouse (proposed by H. Deslandres) ;
Major John Cassells, V.D., J.P., 154 Queen's Drive, Crosshill, Glasgow (proposed by John Dansken) ;
Patrick Sinclair Hardie, M.A., B.Sc., Mathematical Master, 305 Onslow Drive, Dennistoun, Glasgow (proposed by J. McCarthy) ; and
Richard Kerr, F.G.S., Lecturer on Astronomy, Experimental Physics, &c., 13 Ormiston Road, Greenwich, S.E. (proposed by E. W. Maunder).

One hundred and sixty-four presents were announced as having been received since the last meeting, including amongst others :—

T. W. Backhouse, publications of West Hendon House Observatory, No. 2, presented by Mr. Backhouse ; F. H. Bigelow, Eclipse Meteorology and allied problems, presented by the author ; J. Bossert, Détermination des mouvements propres des étoiles, &c. [4 memoirs], presented by the author ; Breslau, Universitäts-Sternwarte, Festschrift zum 90 Geburtstage des Herrn Prof. J. G. Galle, presented by the Observatory ; Cape of Good Hope, Geodetic Survey, vol. 2, Report on re-discussion of Bailey's and Fourcade's surveys, &c. by Sir D. Gill, presented by the Observatory ; Agnes M. Clerke, History of Astronomy during the 19th century, 4th edition, presented by A. & C. Black ; B. Hasselberg, Spectra der Metalle, VI., Molybdän, presented by the author ; E. W. Maunder, Astronomy without a telescope, presented by Witherby & Co. ; Oxford University Observatory, Miscellaneous papers, 1899–1900, presented by the Observatory ; South Kensington, Solar Physics Observatory, The Sun's spotted area, 1832–1900, presented by the Observatory ; S. Tromholt, Catalog der in Norwegen beobachteten Nordlichter, presented by the Christiania Society of Sciences ; C. A. Young, Manual of Astronomy, presented by the author ; 2 photographs of Perrine's Comet, presented by the Astronomer Royal and 4 photographs presented by R. C. Jonnson ; portrait of John Goodricke, presented by W. T. Lynn ; 3 photographs of Presidents of the Society, presented by W. H. Wesley.

Sur la Précision des Mesures Photographiques : Réponse à deux Notes de M. H. C. Plummer. Par M. Loewy.

Dans les *Monthly Notices*, vol. lxi., Supplementary Number, p. 618–628 et vol. lxii., No. 7 (1902 May), p. 506–516, M. Plummer formule des critiques nombreuses sur les conclusions auxquelles je suis parvenu dans mes mémoires relatifs à la précision des coordonnées des astres obtenues à l'aide des mesures effectuées sur leurs images photographiées.

Le dernier numéro (1902 May) ne m'étant parvenu que dans le courant du mois de Juin dernier, il m'a été impossible de présenter à la Société royale Astronomique, dans sa séance de Juin, la réponse que je dois opposer aux allégations de M. H. C. Plummer, allégations qui, je n'hésite pas à le dire, étonneront les astronomes qui ont fait des applications rationnelles de la méthode des moindres carrés.

On peut classer les critiques de M. Plummer en deux catégories : les unes reposent sur des interprétations erronées

des tableaux de nombres publiés dans mes mémoires ; les autres ont le caractère d'affirmations pures et simples, dénuées de toute démonstration, en contradiction d'ailleurs avec la réalité des faits.

Ce qui produit un antagonisme aussi absolu entre les vues de M. Plummer et les miennes, c'est que :

1°. M. Plummer ne semble pas se rendre suffisamment compte du caractère différent des erreurs qui interviennent dans toute étude de précision et qui peuvent être classées en trois catégories : erreurs purement accidentelles—erreurs possédant une permanence plus ou moins prolongée, comparables à des inégalités de courtes périodes et affectant d'une même manière des séries de mesures faites dans de petits intervalles de temps ou dans des conditions physiques susceptibles de varier d'un moment à l'autre—erreurs systématiques constantes.

2°. Il n'envisage, dans sa discussion, qu'un tout petit côté de la question ; il examine entre quelles limites la fréquence relative des erreurs concorde avec les règles bien connues du calcul des probabilités. On n'obtient ainsi qu'un critérium très incomplet dont M. Plummer fait souvent un usage non justifié. En dehors même de toute considération sur les conclusions dans une certaine mesure problématiques fournies par le calcul théorique, les règles qui en découlent ne peuvent se manifester que si les données conduisant aux résidus sont en nombre très considérable. C'est à cette condition seule qu'on peut espérer obtenir dans les résidus, relatifs aux deux premières catégories d'erreurs, une distribution en accord convenable avec les prévisions théoriques. Or, dans l'immense majorité des recherches, quelle que soit leur nature, une telle quantité de données homogènes fait défaut ; néanmoins les conclusions qui s'en dégagent restent exactes bien que le contrôle spécial en question n'y soit pas utilisable.

Sous ce rapport des difficultés se présentent même dans les cas les plus simples. En déterminant, par exemple, la valeur d'une inconnue à l'aide de pointés effectués sur un objet unique, astre ou mire terrestre, on est exposé, par suite des fluctuations de l'équation personnelle ou d'appréciations erronées plus ou moins persistantes sur la position du centre des images stellaires, à affecter une série de lectures successives (10 ou 20) d'une même faible erreur constante, inexactitude qui devient autre pour une seconde série exécutée plus tard.

C'est pour de semblables raisons que les astronomes n'ont qu'exceptionnellement recours à ce procédé de contrôle bien connu auquel il manque presque toujours la base nécessaire.

En examinant la loi de distribution des résidus, M. Plummer pourra se donner la satisfaction illusoire de mettre en doute tout ce qui a été publié à peu près jusqu'à l'heure actuelle par les astronomes les plus illustres qui, je suppose, n'ont pas ignoré les règles du calcul des probabilités.

M. Plummer a tenté d'invalider de cette manière quelques-unes des conclusions de mon second mémoire, mais il sera

démontré un peu plus loin que là, même où un matériel d'observations suffisant a permis de recourir à cette méthode de contrôle dont la rigueur n'est pas absolue, M. Plummer en a fait une application irrationnelle.

Le but que je me proposais dans les mémoires que je viens de publier était d'obtenir l'erreur probable réelle des coordonnées astronomiques tirées des clichés. On sait que jusqu'à présent la nature des opérations multiples exécutées dans la recherche des constantes astronomiques n'a jamais permis d'évaluer à priori le véritable degré d'exactitude du résultat cherché.

Je vais maintenant examiner en détail les objections de M. Plummer et commencer par ses deux critiques principales à l'appui desquelles il a fourni deux tableaux de chiffres empruntés à mes mémoires et qui ont une signification tout autre que celle qu'il leur attribue.

Il a été signalé, dans mon second mémoire, un fait particulier aux observateurs A et B, à savoir qu'il ne se manifeste aucune différence systématique sensible entre les coordonnées des images mesurées par eux, fait qui se produira souvent d'ailleurs pour des opérateurs exercés.

Les tableaux III–2 à III–7, pages 90 et suivantes (circ. n° 9), contiennent 258 ordonnées mesurées dans quatre orientations différentes par l'observateur A et par l'observateur B, et en outre les différences $y_A - y_B$ entre les nombres obtenus pour les deux observateurs. Ces coordonnées sont relatives à 43 étoiles sur lesquelles on a effectué 6 poses successives donnant ainsi 6 images pour chaque astre.

Lorsqu'on prend la moyenne de ces 258 différences $y_A - y_B$, on ne trouve aucun désaccord systématique entre les ordonnées y_A et y_B déterminés par les deux observateurs. La différence moyenne $\frac{yA - yB}{258} = -0''{\cdot}005$, $\pm 0''{\cdot}004$, est seulement de quelques millièmes de seconde d'arc et provient uniquement, ainsi qu'on le voit, d'inexactitudes accidentelles dans les mesures. C'est un fait incontestable. M. Plummer aurait pu, en cette circonstance, appliquer sa méthode de prédilection, puisque, dans ce cas, une des conditions essentielles du calcul des probabilités se trouve réalisée ; on est en effet en présence d'un matériel d'observations homogènes conduisant à un nombre assez respectable (258) d'erreurs de caractère accidentel.

Pour mieux mettre ce point en lumière, on a inscrit, dans le tableau suivant, le nombre réel des erreurs de $0''{\cdot}03$ en $0''{\cdot}03$, selon leur grandeur et le nombre calculé d'après la théorie. On constatera aisément que l'accord entre les deux séries de chiffres est aussi grand qu'il peut et doit l'être, d'après la vraie nature des choses.

On doit s'attendre en effet à trouver dans la réalité un nombre légèrement plus élevé pour les grands écarts, à cause de l'imperfection de quelques images et des difficultés de mesure qu'on

rencontre quelquefois lorsqu'une image est placée près d'un trait du réseau.

Groupement et Nombre des Erreurs suivant leur Grandeur.

De o″·oo à	Réalité.	Diff.	Calcul.	Diff.
0·03	65	72	63	57
0·06	137	48	120	48
0·09	185	31	168	35
0·12	216	19	203	24
0·15	235	9	227	15
0·18	244	5	242	8
0·21	249	3	250	5
0·24	252	0	255	2
0·27	252	2	257	1
0·33	254	4	258	0
0·48	258		258	

Erreur probable d'une différence $y_A - y_B = \pm 0''{\cdot}065$.

Tout astronome, j'en suis convaincu, conviendra qu'il n'y avait ici aucune distinction à admettre entre les modes d'opération des deux observateurs. Mais M. Plummer, qui semble être animé d'une prédisposition systématique à la critique, ne veut pas se rendre à l'évidence.

Voici le procédé auquel il a recours pour jeter un doute sur ce résultat. Au lieu d'examiner la loi de distribution des 258 résidus directement fournis, distribution qui se trouve en harmonie satisfaisante avec le calcul théorique, M. Plummer forme la moyenne des 6 ordonnées des images relatives à une même étoile, de sorte qu'il ramène la discussion, contrairement à la logique de la méthode des moindres carrés, à un nombre de données beaucoup moindre, à l'examen de 43 valeurs seulement, où le hasard aveugle peut se manifester davantage.

Malgré la voie indirecte ainsi suivie, le tableau de chiffres fourni par M. Plummer, *Monthly Notices*, vol. lxii. p. 514, ne peut, étant bien interprété, que confirmer les résultats tirés des tableaux originaux. Pour les 6 ordonnées relatives à 4 étoiles, on y constate des différences $y_A - y_B$ de même sens, tandis que, pour 4 autres étoiles, ces différences sont affectées du signe contraire ; pour les 35 étoiles restantes les signes sont alternés.

Pour un esprit non prévenu, ces deux séries de 4 coïncidences, à cause même du fait qu'elles sont de sens contraires, n'ont rien de surprenant.

M. Plummer considère ces résultats comme anormaux et comme révélant une sorte d'incertitude systématique dans le mode d'opération des deux observateurs. Cette opinion est inadmissible. Des écarts analogues se présentent, comme le

savent tous les astronomes expérimentés, dans les séries de mesures les plus simples effectuées même par un observateur seul, à de courts intervalles, sur les objets les mieux définis. La pratique enseigne à se défier des pointés consécutifs et concordants ; les divergences, qui se manifestent entre les séries séparées par de notables intervalles, donnent lieu au contraire d'espérer qu'on s'est affranchi des influences physiologiques ou physiques qui se modifient avec le temps. Dans le cas qui nous occupe deux pointés ont été faits par chacun des deux observateurs sur chacune des 6 images. Il est dans la nature des choses de rencontrer de temps à autre des permanences dans le signe des résidus.

En ce qui concerne la grandeur de quelques-uns des écarts qui excitent l'étonnement de M. Plummer (étoiles n^{os} 15 et 29), il suffit d'examiner les résultats d'un cliché quelconque, même des mieux exécutés, pour reconnaître que les images de certaines étoiles sont moins bien venues et ne comportent pas des mesures aussi précises que les autres. Il a été de règle de mesurer à nouveau les coordonnées relatives aux résidus notables. Toutes les valeurs publiées correspondent donc à une réalité indiscutable.

La permanence des signes pour l'étoile n° 15 aurait dû être pour M. Plummer l'indice d'une imperfection relative dans une au moins des images de cette étoile. Dans le registre des mesures on trouve en effet les renseignements suivants :

Etoile n° 15, images allongées.

Etoile n° 29, dernière image du groupe I touche presque le trait du réseau.

Dans ma longue carrière j'ai souvent eu à confronter les résultats émanés de diverses sources et, comme M. Plummer, je me suis attaché quelquefois à noter la fréquence relative des erreurs. Mais la pratique m'a montré que la rareté des grands écarts doit être plutôt un motif de défiance, car elle n'est souvent qu'un résultat de la tendance des observateurs à supprimer les mesures que les écarts avec la moyenne leur dénoncent comme les plus défectueuses. Des écarts sept ou huit fois plus grands que l'erreur probable peuvent parfaitement se présenter dans la pratique et ne justifient pas la suppression des observations correspondantes. Il n'arrive pour ainsi dire jamais que l'état physiologique de l'observateur et la qualité des images stellaires présentent l'homogénéité que suppose l'application du calcul des erreurs.

La première et principale objection de M. Plummer est donc dénuée de toute base réelle et, ce qui était à prévoir, le tableau même présenté par lui ne fait que confirmer qu'il n'y a aucune distinction à établir entre les coordonnées émanant des deux observateurs A et B.

Pour des raisons analogues la seconde critique de M. Plummer n'est pas plus justifiée que la précédente. M. Plummer affirme qu'en combinant les coordonnées obtenues dans deux orientations différant de 180°, on élimine d'une manière à peu près complète

les erreurs systématiques et que les résultats ainsi obtenus offrent des garanties d'exactitude très suffisantes.

Il a été établi par mes recherches que, lorsqu'on mesure les coordonnées rectilignes dans une orientation quelconque de la plaque, ces coordonnées se trouvent affectées de deux catégories d'erreurs de mesure dont l'une provient des inexactitudes purement fortuites des pointés, et dont l'autre, d'une nature plus complexe, reste invariable et dépend de plusieurs causes dont les deux principales sont : l'erreur physiologique Es_1 de l'astronome qui ne place pas l'image au milieu des deux fils parallèles du micromètre et celle Es_2 qui provient d'une appréciation erronée du centre du disque qui, même pour les meilleures images, ne présente pas toujours une netteté parfaite.

La première Es_1 ne se manifeste pas ou à peine pour des observateurs habiles ; son influence s'éliminerait toutefois par la combinaison des mesures faites coup sur coup dans deux orientations différant de 180° ; la seconde Es_2 est indépendante de l'observateur et provient, comme je l'ai expliqué (circ. n° 9, p. 9), de tout un ensemble de causes physiques et optiques, du manque d'homogénéité de la couche de gélatine, de la définition imparfaite des images, du microscope employé aux mesures et de sa mise au foyer plus ou moins exacte, des conditions d'éclairage, etc. . . . Cette seconde source d'erreurs, dont M. Plummer nie l'existence, ne peut être évitée et les séries de mesures faites dans des orientations différentes de la plaque en subissent toujours plus ou moins l'influence.

Pour mettre ces conclusions en défaut, M. Plummer emploie les mêmes tableaux déjà cités et en fait un nouvel extrait qui le conduit à des conclusions contraires à celles qui se dégagent des nombres originaux quand on y applique directement les méthodes du calcul des probabilités.

Dans les tableaux de mon second mémoire on trouve, pour chaque étoile, 8 ordonnées mesurées dans 4 orientations différentes par chacun des deux observateurs. Pour mettre en lumière l'influence et la grandeur de chacune des deux erreurs systématiques Es_1 et Es_2, on a conclu les erreurs probables par deux méthodes différentes. La solution I a été obtenue en calculant pour chacune des 43 étoiles la moyenne M des 8 ordonnées respectives correspondantes :

$$M = \frac{y_0^A + y_{90}^A + y_{180}^A + y_{270}^A + y_0^B + y_{90}^B + y_{180}^B + y_{270}^B}{8},$$

puisqu'il n'y a aucune différence à faire entre le mode de mesure des 2 observateurs ; on a ensuite formé les résidus en comparant chacune des 8 ordonnées à leur moyenne M et de l'ensemble des résidus on a déduit les erreurs probables indiquées sous le titre solution I dans le tableau donné plus loin.

Pour la solution II, on a d'abord pris, pour chaque observateur A et B, les moyennes $\frac{y_0+y_{180}}{2}$, $\frac{y_{90}+y_{270}}{2}$ et, ensuite, les

différences $\frac{y_0+y_{180}}{2}-\frac{y_{90}+y_{270}}{2}$, différences dès lors indépendantes de l'erreur physiologique Es_1. L'ensemble de ces résidus $\frac{y_0+y_{180}}{2}-\frac{y_{90}+y_{270}}{2}$ à conduit aux erreurs probables figurant sous le titre solution II dans le tableau ci-après :

	Solution I.		Solution II.	
	Groupe I.	Groupe III.	Groupe I.	Groupe III.
Première image	$Ep = \pm 0''095$	$Ep = \pm 0''097$	$Ep = \pm 0''088$	$Ep = \pm 0''084$
Deuxième image	$\pm 0''099$	$\pm 0''097$	$\pm 0''084$	$\pm 0''082$
Troisième image	$\pm 0''107$	$\pm 0''104$	$\pm 0''097$	$\pm 0''095$
Moyenne...	... $\pm 0''100$	$\pm 0''099$	$\pm 0''090$	$\pm 0''087$

Comme on le voit, les deux solutions révèlent d'un commun accord l'existence d'un erreur systématique Es, puisque leurs valeurs numériques dépassent notablement l'erreur accidentelle Ea d'une moyenne reposant sur deux pointés.

En prenant la moyenne entre les ordonnées respectives différant de 180°, on a rendu la seconde solution indépendante de l'erreur physiologique Es_1 ; elle ne renferme dès lors que l'erreur physique Es_2. La première solution, au contraire, où l'on a supposé l'indépendance absolue des résidus dans les diverses orientations, met en évidence la totalité de l'erreur systématique $Es = \sqrt{(Es_1)^2+(Es_2)^2}$.

Le peu de différence qui existe entre les nombres de la solution I et ceux de la solution II prouve que l'erreur physiologique Es_1, pour les deux observateurs considérés, ne joue qu'un rôle tout à fait secondaire.

M. Plummer, par une méthode purement inductive, tente de mettre en doute ces conclusions. Il forme la moyenne $M = \frac{y_0+y_{90}+y_{180}+y_{270}}{4}$ des 4 ordonnées pour chaque observateur et pour chaque image, et il compare chacune des 4 ordonnées individuelles à leur moyenne ; il obtient ainsi pour chaque image, dans chaque orientation, 43 résidus dont il forme la moyenne générale. C'est ainsi qu'il a établi (*Monthly Notices*, p. 513, vol. lxii.) un tableau dont nous allons examiner maintenant la véritable signification.

Pour mieux faire comprendre la discussion qui va suivre il est utile de fournir ici quelques courtes explications sur la manière dont les mesures ont été effectuées.

Les ordonnées des images ont été mesurées dans une position déterminée du cliché, d'une manière ininterrompue par le même observateur dans l'espace de quelques heures, par conséquent dans les mêmes conditions physiques d'éclairage, de mise au foyer, etc. . . . En outre, comme il s'agit d'images de même espèce, images à contours normaux, on doit s'attendre à ce que l'erreur

physique Es_2 ait pu conserver en majeure partie le même signe dans chaque position. Mais cette erreur physique Es_2 aura en général des valeurs différentes et des signes quelconques pour toute autre orientation.

Chaque série individuelle de 43 résidus se trouvant affectée d'une erreur systématique Es, il en résulte naturellement que chacune des 4 moyennes M_0, M_{90}, M_{180}, M_{270} de ces quatre séries de 43 résidus doit différer sensiblement de zéro. Lorsqu'on forme ensuite la moyenne M de ces quatre nombres moyens, $M = \frac{m_0 + m_{90} + m_{180} + m_{270}}{4}$, et les différences $M - m_0$, $M - m_{90}$, $M - m_{180}$, $M - m_{270}$, on doit généralement rencontrer deux de ces différences affectées du signe + et les deux autres du signe −, puisque la somme de ces quatre valeurs doit être égale à 0 ; et, contrairement à la thèse de M. Plummer, des compensations forcées se produiront aussi bien pour les positions différant de 90° que pour celles différant de 180°. C'est cette conclusion si élémentaire qui n'a pas été aperçue par M. Plummer.

Plaçons maintenant sous les yeux du lecteur les nombres moyens du tableau de M. Plummer (p. 513 des *Monthly Notices*, vol. xlii.).

	Observateur A.				Observateur B.			
	0°.	180°.	90°.	270°.	0°.	180°.	90°.	270°.
Moyennes	+0″·096	−0″·108	+0″·072	−0″·059	+0″·073	−0″·042	+0″·035	−0″·066

Ou en écrivant ces nombres dans un ordre un peu différent on a aussi :

	Observateur A.				Observateur B.			
	270°.	0°.	180°.	90°.	270°.	0°.	180°.	90°.
Moyennes	−0″·059	+0″·096	−0″·108	+0″·072	−0″·066	+0″·073	−0″·042	+0″·035

On y constate sans difficulté que des compensations complètes se présentent pour les positions différant seulement de 90° ; pour l'observateur B en particulier les compensations sont encore plus complètes pour les positions différant de 90° (270 et 0) (180 et 270) que pour celles de 180°.

En résumé le tableau de M. Plummer, qui est purement inductif, ne peut fournir que des renseignements généraux complètement en accord d'ailleurs avec les conclusions fournies par l'application directe de la méthode des moindres carrés. Il indique que les opérations faites dans une seule orientation sont affectées d'erreurs systématiques qui se compensent en partie si on les combine avec des mesures réalisées dans une autre orientation quelconque. Mais, pour se rendre compte d'une manière précise de la provenance et de l'importance des erreurs systématiques, il fallait exécuter une analyse directe, comme je l'ai fait dans le second mémoire, en recherchant l'une après l'autre les diverses causes d'erreurs systématiques influant sur les résultats. Il a été

ainsi établi plus haut, qu'après avoir formé les différences $\frac{y_0+y_{180}}{2}-\frac{y_{90}+y_{270}}{2}$, différences indépendantes de l'erreur physiologique Es_1, les résidus décèlent la présence d'une erreur systématique Es_2, pour laquelle on a adopté la valeur $\pm 0''.073$, supérieure à l'erreur physiologique Es_1 qui, à cause de sa faiblesse, pouvait être considérée comme négligeable. S'il n'en avait pas été ainsi, on aurait fait entrer dans les formules l'effet combiné des deux éléments Es_1 et Es_2. Il est loin de ma pensée de prétendre que l'erreur physiologique Es_1 n'existe pas pour d'autres observateurs ; elle se manifestera probablement d'une manière sensible dans un certain nombre de cas. Pour la mettre en lumière et pouvoir en tenir compte, on n'aura qu'à suivre la voie qui vient d'être indiquée, c'est-à-dire comparer les nombres résultants des solutions I et II.

Il convient maintenant d'examiner les affirmations dans lesquelles s'aventure M. Plummer.

Il en est une que je dois relever d'abord comme dénuée de tout fondement et de plus peu courtoise pour les deux observateurs chargés des mesures des clichés relatifs à la planète *Eros*. M. Plummer a abusé d'une remarque faite à la page 11 de la circulaire n° 9 : "*En ce qui concerne le cliché P, quelques incertitudes très légères se sont révélées au début des mesures, où les observateurs A et B n'avaient pas la sûreté d'une pratique prolongée, mais ultérieurement pour tous les deux l'erreur physiologique Es_1 s'est complètement évanouie.*"

Les deux observateurs dont il est question sont des observateurs excellents et, à l'exception de leurs deux ou trois premières séances au début des opérations consacrées au cliché P, toutes les nombreuses mesures ultérieures ont été exécutées par eux dans des conditions de précision qui peuvent être difficilement surpassées. Les quelques écarts qui se trouvent dans les séries de résidus n'étonneront aucun astronome expérimenté ; si ces écarts y manquaient on serait en droit de soupçonner l'effet de suppressions arbitraires.

M. Plummer déclare magistralement que les données que j'ai utilisées ne sont pas suffisantes pour déterminer les constantes qui entrent dans mes formules. C'est encore une affirmation, faite très à la légère.

La plupart de ces constantes ont été évaluées directement. C'est ainsi que l'erreur accidentelle Ea a été déduite de séries considérables de pointés, de même que l'erreur de pointés sur les traits du réseau a été déterminée directement, les images stellaires n'y intervenant pas. Il suffit pour fixer la grandeur de ces constantes d'étudier un ou deux clichés ; mais ici ces expériences ont été multipliées. Ces deux constantes sont donc fixées à priori et d'une manière absolument indépendante les unes des autres. Je ne crois pas qu'il existe un observatoire s'occupant de photographie astronomique de précision, où l'on n'ait pas pratiqué ce genre d'expériences et où on ne connaîtrait pas d'une

manière sûre la valeur numérique de ces constantes si faciles à apprécier.

Pour mettre en évidence l'accroissement de l'erreur probable provenant de la dégradation des disques stellaires, relative aux deux grandeurs-limites, les clichés suivants ont été utilisés :

1900, 23 Octobre, 58 étoiles, à 4 images.
1901, 13 Février, 83 „ 4 „
1900, 19 Octobre, 82 „ 3 „
1900, 25 Décembre { 1re pose : 43 étoiles à 3 images.
2me „ 43 „ 3 „
3me „ 43 „ 3 „

C'est un matériel d'observations bien supérieur à celui qui serait nécessaire pour l'évaluation des deux constantes dont dépend l'augmentation de l'erreur probable. Un seul cliché, réalisé dans des conditions d'exactitude moyenne, contenant quelques centaines d'étoiles jusqu'à la 12me grandeur environ et présentant deux ou trois images pour chaque astre, permettra de vérifier d'une manière complète ces conclusions si importantes au point de vue de la pratique. Au lieu de se contenter de produire des négations pures et simples, M. Plummer ferait mieux d'emprunter à quelque observatoire un ou deux clichés et d'effectuer sur les coordonnées mesurées quelques-unes des recherches dont il met en doute les résultats. En groupant, par exemple, les différences entre les ordonnées de deux images voisines d'un même astre suivant l'ordre de grandeur des images stellaires, il verrait de suite que l'erreur probable croît dans une proportion considérable à mesure qu'on se rapproche de la limite de visibilité des étoiles photographiées. Il constaterait en même temps que les mesures effectuées sur les disques les plus faibles sont aussi précises que celles qui se rapportent aux images les plus visibles. Il serait ainsi amené à se convaincre que l'accroissement de l'erreur probable, spécial aux deux dernières grandeurs-limites, est dû uniquement à la dégradation des disques qui n'apparaissent qu'imparfaitement sous l'action de faisceaux lumineux très faibles. Toutes ces conséquences lui sauteraient aux yeux immédiatement. L'expérience lui ferait de même toucher du doigt l'action qui tient à la déformation irrégulière de la gélatine, action qui déplace à la fois les traits du réseau et les images stellaires. Je considère comme une des plus importantes conclusions de mes dernières recherches d'avoir démontré que ces inexactitudes de nature purement physique, indépendantes des opérations de mesure, jouent un rôle capital dans la détermination des coordonnées rectilignes ; et que, si l'on n'a pas pris les précautions indiquées pour atténuer ces causes perturbatrices dans l'exécution des clichés, il devient inutile par suite de multiplier les mesures, aussi bien au point de vue du nombre des pointés que de celui des orientations. Les deux

formules données page 109, circ. n° 9, permettent d'évaluer séparément l'influence des opérations de mesure et celle des causes physiques.

M. Plummer (*Monthly Notices*, vol. lxi. page 626, §12) établit la règle absolue qu'il faut adopter pour unité de mesure et comme seule base de détermination des coordonnées rectilignes la moyenne des lectures obtenues dans deux orientations de la plaque différant de 180° ; et il trouve peu profitable d'effectuer des mesures dans des orientations plus variées, mais il ne donne aucun moyen d'évaluer l'exactitude ni dans un cas ni dans l'autre. Ces opinions sont insoutenables.

Dans chaque recherche individuelle les précautions à prendre sont dictées par la précision qu'on désire atteindre et, selon les circonstances, il sera judicieux d'exécuter les travaux de mesure dans une, deux ou quatre orientations différentes du cliché et même de les faire répéter encore une seconde fois. Lorsqu'il s'agit d'une vaste entreprise, telle que la Carte photographique du Ciel, par exemple, où l'on se propose d'obtenir les coordonnées photographiques de millions d'astres, le degré d'exactitude réclamé ne sera pas le même que celui exigé dans la détermination des parallaxes stellaires ou des positions de la planète *Eros*. Les formules publiées ont été établies dans le but de pouvoir se rendre compte à priori de l'ensemble des opérations de mesure à accomplir pour arriver au degré d'approximation désiré.

Comme les opinions émises par M. Plummer pourraient jeter quelque discrédit sur les travaux relatifs à la Carte du Ciel qui, dans plusieurs observatoires, reposent sur des observations effectuées dans une seule orientation de la plaque, il est nécessaire de montrer que ce mode d'opération comporte un degré d'exactitude tout à fait en rapport avec le but poursuivi. Dans cette entreprise, on a en vue d'obtenir à l'aide de la photographie les positions de quelques millions d'étoiles avec la même précision que celle fournie par les meilleures observations méridiennes. Conformément aux conventions établies, les coordonnées photographiques reposent, dans cette œuvre, sur la moyenne des mesures effectuées sur deux images de chaque étoile. Voici le renseignement que fournissent à cet égard les formules développées, en désignant par Ep^{ch} l'erreur tenant à la constitution de la couche sensible et aux actions chimiques qui interviennent dans la production des images et par Ep^{m} celle provenant uniquement des opérations de mesure : pour 1 orientation . . . $Ep^{ch} = \pm 0''{\cdot}098$. $Ep^{m} = \pm 0''{\cdot}094$. $Ep^{totale} = \pm 0''{\cdot}136$. Pour 2 orientations . . . $Ep^{ch} = \pm 0''{\cdot}098$. $Ep^{m} = \pm 0''{\cdot}067$. $Ep^{totale} = \pm 0''{\cdot}118$.

Ainsi, la précision atteinte dans les deux cas ne diffère pas sensiblement, à cause de l'action considérable des erreurs physiques qui ne s'éliminent pas quel que soit le nombre de mesures effectuées. On constate de plus que, dans les deux cas, l'exactitude obtenue dépasse celle que réalise la moyenne de plusieurs observations méridiennes. En résumé : l'emploi de deux orientations exige une augmentation notable du travail, mais offre

l'avantage de signaler immédiatement les grosses méprises qui peuvent se glisser dans l'exécution des mesures ; en opérant dans une seule orientation on économise des labeurs notables, mais on doit, pour vérifier les coordonnées, attendre que la seconde série de plaques prévue ait été réduite.

Cette conclusion acquiert encore plus de poids si l'on considère qu'à ces inexactitudes s'ajoute l'erreur bien plus grande provenant des positions des étoiles de repère destinées à la détermination des constantes des clichés. En effet une recherche, accomplie à l'occasion du premier volume du catalogue photographique (coordonnées retangulaires) de la zone de Paris, a conduit à évaluer à $\pm 0''.26$ au minimum l'incertitude provenant de cette source et relative à l'une ou l'autre des deux coordonnées du centre de la plaque. Dans cette publication les constantes des clichés sont déduites à l'aide des positions de 21 étoiles de repère tirées de tous les catalogues existants (de 6 environ, en moyenne). Toutefois cette incertitude peut être notablement amoindrie si l'on effectue à l'époque actuelle des observations méridiennes particulièrement consacrées à ce but. Quoiqu'il en soit sous ce dernier rapport il est permis d'affirmer que dans l'état actuel des choses l'exactitude d'une coordonnée stellaire rapportée au centre du cliché par l'un ou l'autre des deux procédés de mesure indiqués dépasse dans une large proportion la précision exigée par le plan de travail. Ainsi qu'on le voit elle est au moins équivalente à celle fournie par une quarantaine de positions méridiennes empruntées aux catalogues d'étoiles actuellement connus.

Les deux procédés sont donc à des titres différents également recommandables. M. Plummer aurait dû, avant de formuler ses critiques, supposer que les directeurs d'observatoires, qui ont la science et la pratique de ces travaux, avaient eu quelque motif de choisir le premier procédé pour des raisons ignorées par lui.

Il me reste encore une dernière remarque à présenter au sujet d'une des critiques de M. Plummer, qui repose uniquement sur une interprétation erronée de ma pensée. Il s'imagine que j'ai conseillé aux astronomes d'opérer, pour tous les cas, dans quatre orientations de la plaque. J'ai seulement dit : Si l'on veut atteindre un très haut degré de précision dans la détermination des coordonnées, il faut agir de la sorte ; et, en outre, j'ai fait ressortir, dans les derniers mémoires, que ce surcroit de travail ne saurait porter ses fruits si l'on n'a pas exécuté les clichés dans des conditions telles que les erreurs physiques aient été amoindries dans une proportion considérable, c'est-à-dire en faisant deux ou trois groupes de poses multiples, séparés les uns des autres par un intervalle de plusieurs minutes d'arc.

Les recherches expérimentales ne peuvent inspirer confiance qu'à la condition qu'on soit parvenu à fixer d'avance l'exactitude que possèdent les données fondamentales dont dépend le résultat final. Dans les observatoires particulièrement consacrés à la

photographie céleste, où on produit des centaines de clichés de même nature, il importe de connaître le véritable degré d'approximation avec lequel on peut tirer d'une de ces plaques les coordonnées des astres photographiés. Cette enquête si nécessaire est, heureusement, quoi qu'en pense M. Plummer, très facile à réaliser ; il suffit en suivant à peu près la voie indiquée d'étudier tout au plus deux ou trois clichés pour se rendre compte de l'influence numérique de chacune des diverses causes d'inexactitude mentionnées et dont l'ensemble altère les coordonnées conclues. Selon la constitution des clichés on trouvera, bien entendu, des valeurs différentes pour les erreurs probables correspondantes, mais on vérifiera toujours sans peine la réalité des sources qui les ont fait naître.

En utilisant les données fournies dans mon premier mémoire, circ. n° 8, M. Plummer a fourni, d'après son procédé, une appréciation de l'erreur probable d'une ordonnée mesurée dans deux orientations. Il trouve ainsi (*Monthly Notices*, vol. lxi. p. 627) $\pm 0''.060$. En présentant un tel résultat M. Plummer a omis de tenir compte de tout un ensemble de causes physiques dont la valeur numérique est maintenant connue. L'erreur réelle dans le cas considéré (cliché P, 3 images, 2 pointés, 2 orientations) est $\pm 0''.10$, et encore ce nombre ne s'applique qu'aux images précédant d'une grandeur la limite de visibilité. M. Plummer suit dans ce cas les mêmes errements que les astronomes qui, durant une partie notable du siècle écoulé, ont calculé des erreurs probables tellement en désaccord avec la réalité qu'ils ont failli compromettre l'application de la belle méthode des moindres carrés.

Il m'a paru nécessaire de répondre aux critiques de M. Plummer, critiques qui pouvaient influencer l'appréciation des lecteurs peu familiers avec les études spéciales dont il est question. Mais je ne continuerai plus une polémique qui me paraît désormais dépourvue de toute utilité.

On the Accuracy of Photographic Measures. Third Note: Reply to M. Loewy. By H. C. Plummer, M.A.

By the courtesy of the Secretaries I have been permitted to see the preceding paper in proof, and it may be convenient to have my reply at once.

In one thing at least I am in the heartiest agreement with M. Loewy. It would be quite unprofitable to prolong the present controversy. I shall therefore not be misunderstood if I do not discuss the specific points to which M. Loewy refers.

With the advantage of having read M. Loewy's reply, I should have been glad to take the opportunity to revise my

former notes. But after a most careful comparison of his paper, paragraph by paragraph, with what I have already published, I cannot admit that he has shown my conclusions to be faulty. I naturally hope that anyone who has the leisure to examine both M. Loewy's paper and my own with the same attention will arrive at the same opinion. In particular, as regards §§ 14 and 12 of my second note, to the consideration of which M. Loewy devotes fully one-half of his reply, I am perfectly willing that my words should stand as they were written.

Fortunately, with one exception, M. Loewy raises no question as to my facts or my figures. It is from the inferences based thereon that he dissents. The inferences must be left to speak for themselves. The one question of fact may be examined. I stated, subject to correction by M. Loewy, that the complicated formulæ and very important conclusions of his first two memoirs were based on the evidence of a single plate. He now gives a list of four plates. But the first two, if they can be identified by their dates, were devoted to the study of the influence of trails. The results of this research form the subject of two distinct papers, and really lie outside the limits of the discussion, though they receive passing mention in the second of the two memoirs in question (*Circulaire*, No. 9, p. 32). Moreover, the fourth plate is only introduced in the third memoir. Thus M. Loewy only confirms me in my belief that too much weight was attached to the evidence of the single plate P.

I am sure that it was far from M. Loewy's intention to do any injustice to my views. Yet I confess that I should have been better satisfied had he in some instances quoted my actual words. Thus I have looked in vain for the "négations pures et simples" (p. 11) to which the passage which follows can be applied. And similarly, if he had quoted the terms of my reference to the method of measuring in one orientation, the reader would have been enabled to judge for himself whether I had necessarily overlooked the other side of an important question.

In conclusion, I venture to reaffirm the limitations which I have imposed on myself throughout this discussion, and which ought not to be lost from view. Both my former notes were confined to a consideration of the accuracy of the *measures* of photographic images, and were not concerned with the accuracy of the positions of the images themselves.

University Observatory, Oxford:
1902 *November* 20.

On the Images formed by a Parabolic Mirror. Second Paper: Influence on the Measurement and Reduction of a Photograph. By H. C. Plummer, M.A.

1. The relative merits of the mirror and the objective corrected for photographic rays were keenly debated at the first conference of the Astrographic Chart Committee. The wisdom of the decision to reject the reflector for the purpose in view has not, and need not, be questioned. On the other hand, the value of the instrument for certain branches of work cannot be denied, and it is probable that the reflector will be used in the future even more extensively than at present. Already a large store of photographs has been accumulated, and it is important to ascertain how far precise results can be obtained from them. At present there seems to be little material for estimating the degree of accuracy with which reflector photographs can be measured. Though confessedly imperfect in certain directions, this note seems to show that very accurate results cannot be expected from the measurement of at least one important class of photographs of this kind.

2. In accordance with a suggestion by Professor Turner a direct comparison has been made between a photograph taken with a parabolic reflector and a plate taken at the University Observatory, Oxford, for the purpose of the Astrographic Chart Catalogue. The work gave rise to a geometrical study of the aberration of the mirror, the results of which have been given in a former paper.* A photograph of the region containing the nebula $\frac{H}{g}$ V 15 *Cygni* was taken by Dr. Isaac Roberts on 1896 October 10 with an exposure of $2^h\ 18^m$, the instrument employed being his parabolic mirror of 20-inch aperture and 98-inch focal length. The region is practically coincident with that covered by a plate already taken at Oxford for the Astrographic Catalogue. In response to a request for a copy of his photograph, Dr. Roberts very kindly sent to the University Observatory a glass positive. A contact print of this copy was made on another plate, on which a réseau was also impressed. This final copy will be designated Plate A, and it is evident that, as a representation of the sky, it must be considered inferior to the original negative. Not only is the risk of distortion of the film introduced in the process of developing unréseaued plates, but the danger arising from want of perfect contact between the plates in the process of copying† has also to be borne in mind, especially when, as in this case, neither the original negative nor the intermediate positive is on plate glass. It is clear then that the material used in the discussion is drawn from a source which may be distinctly inferior to the original photograph, but the

* *Monthly Notices*, vol. lxii. p. 352.
† Cf. *Monthly Notices*, vol. lix. p. 462.

opinion may be confidently expressed that the difficulties which do present themselves in the consideration of the measures are certainly not due in any great degree to effects arising from this fact.

3. With the Plate A is to be compared the Oxford plate 1571 taken by Mr. Bellamy on 1900 August 22. It bears three exposures of 6 minutes, 3 minutes, and 1 minute respectively, and the approximate position of the plate centre is R.A. $20^h\ 42^m$, Decl. $+30°$. This plate will henceforth be designated Plate B. The positions of the longest exposures had been measured by Mr. H. F. Mullis in the case of those stars, 872 in number, which showed three images. Apparently the centres of both plates are so nearly the same that the measures in the two cases may be directly connected by means of linear formulæ. Now if x_1, y_1 are the coordinates of a star on Plate A (referred to the corner of the réseau as origin) and x_2, y_2 the coordinates of the same star on Plate B, it is found by identifying four particular stars on both plates that if

$$x'_2 = -0\cdot727\ x_2 + 0\cdot010\ y_2 + 22\cdot040$$
$$x'_2 = +0\cdot010\ x_2 + 0\cdot727\ y_2 + 1\cdot420$$

then x'_2, y'_2 do not differ greatly from x_1, y_1. The transformation indicates (1) a reversal due to the fact that one photograph is taken with a reflector, the other with a refractor; (2) a change of scale due to the fact that the focal lengths of the two telescopes are in the ratio of 8 : 11 very nearly; (3) changes of orientation and of origin which are without any importance because they depend entirely on the way in which the glass positive and the réseau were placed in the making of Plate A.

4. The region common to the two plates A and B contains sixty stars which are to be found either in the Cambridge or the Leiden zones of the A.G. Catalogue. By adopting Plate B as a standard for direct comparison, we are independent of the catalogue places of these stars. Nevertheless the direct reduction of Plate A would be based on these stars, and it is therefore interesting to confine the comparison in the first instance to their coordinates. Accordingly, x'_2, y'_2 were calculated from the measures on Plate B, and x_1, y_1 were measured on Plate A. The constants of the linear formulæ

$$ax_1 + by_1 + c = x_1 - x'_2$$
$$dx_1 + ey_1 + f = y_1 - y'_2$$

were then calculated by Mr. Dyson's method,* the use of which is warranted by the number of stars. The resulting values are:

$$a = +0\cdot00134\ ; \quad d = +0\cdot00097$$
$$b = -0\cdot00049\ ; \quad e = +0\cdot00165$$
$$c = -0\cdot0350\ ; \quad f = -0\cdot0334$$

* *Monthly Notices*, vol. lv. p. 61.

The fifth and sixth columns of the following table give the residuals derived from this solution in the sense

$$\delta x = x_1 - x'_2 - (ax_1 + by_1 + c)$$
$$\delta y = y_1 - y'_2 - (dx_1 + ey_1 + f)$$

The scale is such that ·001 = 0″·4 nearly. The stars whose numbers, given in the first column, lie between 11000 and 12000 are to be found in the Cambridge zones ; those whose numbers lie between 8000 and 9000 belong to the Leiden zones. The third and fourth columns contain my own measures on Plate A.

TABLE I.

Cat. No.	Mag.	x_1	y_1	δx	δy	(δx)	(δy)
11640	9·5	20·444	4·718	−·001	+·001	−·002	−·001
11654	,,	19·010	8·755	−·001	·000	−·001	·000
11691	,,	15·734	5·285	·000	−·001	+·003	+·001
11714	,,	14·107	12·760	·000	−·003	+·003	−·001
11718	,,	13·600	12·572	·000	·000	+·003	−·001
11751	,,	10·504	10·067	−·005	·000	−·001	−·002
11787	,,	7·990	6·625	−·001	−·003	−·004	−·003
11650	9·4	19·910	9·086	−·003	·000	·000	−·003
11676	,,	16·361	6·744	·000	+·001	·000	+·004
11689	,,	15·923	13·514	+·003	+·003	+·002	−·002
11753	,,	10·000	4·946	+·004	−·003	+·003	+·003
11832	,,	4·072	6·144	+·005	−·001	+·004	−·002
11636	9·3	21·178	6·802	·000	·000	·000	·000
11678	,,	16·394	12·513	+·003	·000	·000	−·002
11628	9·2	21·676	7·706	+·001	+·001	+·002	+·001
18414	,,	21·090	14·916	+·004	−·001	−·004	+·002
11659	,,	18·748	11·733	+·004	·000	+·001	−·001
11741	,,	11·326	11·745	−·002	·000	+·001	−·002
11776	,,	8·752	8·777	+·002	·000	+·002	+·001
11824	,,	4·993	13·095	+·002	+·001	+·007	−·003
11835	,,	3·889	8·776	+·002	+·003	−·002	+·002
11694	9 1	15·511	10·858	−·001	+·001	·000	−·001
8451	,,	15·457	16·695	−·002	+·001	−·003	+·004
11704	,,	14·745	7·359	−·002	+·001	+·001	·000
11766	,,	9 391	9·322	−·001	·000	−·001	·000
8500	,,	7·753	20·136	−·002	+·005	−·002	·000
8512	,,	5·576	16·948	−·002	−·003	·000	+·001
11819	,,	5·381	8·503	+·001	+·002	·000	−·003
8419	9·0	20·636	14·980	+·002	−·002	−·002	+·004

Cat. No.	Mag.	x_1	y_1	δx	δy	(δx)	(δy)
8421	9·0	20·275	17·406	+·002	−·001	·000	+·001
11682	„	16·144	5·301	−·003	−·002	−·001	−·002
8453	„	15·130	15·584	+·004	+·001	+·002	+·001
11702	„	14·907	10·042	−·001	−·002	+·001	−·001
11707	„	14·637	10·054	−·001	−·002	+·003	−·003
8462	„	13·015	19·227	−·003	+·002	−·001	+·001
8472	„	11·450	15·765	·000	−·001	·000	·000
11748	„	10·657	11·269	+·001	·000	·000	−·001
11757	„	9·874	8·535	−·002	−·001	·000	+·003
8501	„	7·587	17·296	−·003	−·001	−·003	+·003
8519	8·9	4·717	17·185	−·001	·000	·000	−·002
11831	„	4·244	10·410	+·001	−·001	+·003	+·001
8482	8·8	10·859	17·055	−·002	+·001	·000	+·001
11830	„	4·360	6·075	−·002	−·002	+·002	−·005
8447	8·7	15·885	14·906	+·003	−·001	+·002	−·001
8502	„	7·444	19·762	+·001	−·003	+·005	+·001
11663	8·5	18·318	9·546	−·001	−·002	+·001	−·002
8456	„	14·273	16·791	+·003	+·002	·000	+·003
11717	„	13·662	13·464	−·002	−·004	+·002	−·005
11810	„	6·230	7·867	+·003	·000	−·001	·000
11648	8·3	20·059	10·055	+·002	·000	−·002	−·002
11701	„	14·879	6·942	+·003	−·001	+·002	+·003
11826	„	4·726	7·598	+·002	+·003	−·001	−·001
11809	8·1	6·575	13·449	−·002	−·001	·000	−·001
11755	8·0	9·913	4·570	+·003	−·005	·000	·000
11829	„	4·657	11·400	−·001	−·001	−·001	−·003
8513	7·7	5·180	19·606	+·001	−·001	+·001	−·001
8496	7·5	8·415	14·686	·000	+·001	·000	−·001
8518	7·0	4·752	15·754	+·002	−·001	−·002	+·002
8418	6·8	20·756	19·176	−·008	−·005	−·002	−·007
11715	4·4	Not measured					

5. The measures were made, according to the uniform practice at Oxford, in both the direct and the reversed positions of the plate. The non-circular form of the images on a photograph taken with a parabolic mirror makes it necessary to adopt some precise rule for setting on an image. The choice of such a rule involves a problem of no small difficulty and importance. The question presented itself in the course of reductions of photographs of Eros taken at Greenwich with the 30-inch reflector of the Thompson Equatorial * and a systematic change

* *Monthly Notices*, vol. lix. p. 399.

of procedure is recorded. Unfortunately it is more easy to formulate a rule than to follow it, and unless an obvious point, as, for instance, the centre in the case of a disc, is chosen as the mark on which to point in measuring, it is exceedingly difficult to adhere to a consistent standard and to remain unaffected by subjective influences which vary with time and with the aspect under which an image is seen. In the present case the exposure of the reflector photograph was very long in comparison with that of the other plate, and therefore the images on it are apparently well developed—*i.e.* the fainter parts are not lost through under-exposure. The rule adopted was to set on that point of the axis of symmetry corresponding to the greatest width of the image. This point is not the focus for rays reflected at the centre of the mirror, but was chosen in the hope that it would give consistent results. and that it would not altogether fail even in the case of a star too faint to make a full impression. The drawback to this course lies in the fact that the maximum width of an image is by no means well marked, there being two ordinates of nearly equal length.* If, however, a setting is made on a point midway between these ordinates, there will be practically no distortion according to the geometrical theory, though the scale value will be that corresponding to a virtual focal length $f \sec v$, where f is the true focal length and $2v$ the angle subtended by the aperture of the mirror at the focus. Were it desired to find a scale value corresponding to the true focal length, it would be necessary to set on a point which is very near the extremity of the *complete* image in the direction of the centre of the field. But this is not possible, because this point lies outside the photographic image, the rays which should be incident on the centre of the mirror being intercepted by the plate itself.† In all this, however, it is supposed that the plate is accurately adjusted at the focus, and normal to the axis of the mirror. Now neither of these conditions is apparently fulfilled by the plate with which we have to deal. There are no point images at the centre of the plate ; on the contrary, the smallest image exceeds 20″ in diameter. Nor is any marked symmetry of the field apparent about a centre which can be located with any certainty. Other causes, among which must be reckoned imperfections in the figure of the reflector, inequalities in the driving, variations of refraction, &c. conspire to produce images of a character very difficult to measure. On the whole the residuals appear more satisfactory than might have been anticipated.

6. In order to ascertain whether the plan of measurement just explained offered any advantage over bisection of the image a second series of measures was made by Mr. E. A. Gray, an observer possessing experience and skill in the measurement of astrographic plates and unbiassed by preconceived ideas as to the character of the images formed by a reflector. His measures

* See vol. lxii. p. 356.

† Vol. lxii. p. 365.

were treated in the same way as my own, and the residuals (δx) and (δy) are given in the seventh and eighth columns of the preceding table. The solution gave the following constants :

$$a = +0''.00119 ; \quad d = +0''.00086$$
$$b = -0''.00066 ; \quad e = +0''.00149$$
$$c = -0''.0316 ; \quad f = -0''.0294$$

Between this solution and that of §4 there are differences

$$+0''.00015x + 0''.00017y - 0''.0034 ,$$

and

$$+0''.00011x + 0''.00016y - 0''.0040$$

These expressions represent the systematic differences between my own and Mr. Gray's measures. It is evident that they do *not* correspond to a uniform change of scale value with symmetry about a point of the plate, as should be the case according to the way in which they have been obtained. Their approach to equality indicates rather a uniform strain, as it were, perpendicular to a line whose equation is approximately

$$x + y - 23 = 0$$

or very roughly a diagonal of the plate. No reason suggests itself for this peculiarity.

7. The sums of the four series of residuals irrespective of signs place in order of increasing accuracy δx, (δy), (δx), and δy, the sums being in fact ·119, ·108, ·092, ·085 respectively. It may be inferred that both coordinates can be measured with equal precision, and it cannot be said, at present, that either method of measuring possesses any advantage over the other. The probable discordance in a single coordinate may be taken to be $0''.6$. This value and the evidence that a discordance ought very rarely to exceed $2''$ seem more satisfactory than might have been expected. For this reason the preceding table has been given in full. It must be remembered that these figures include not only the errors of the original reflector photograph, but also errors in the measures of the Oxford plate * and any faults possibly introduced in the double process of reproduction. On the other hand these figures apply only to a range of star-magnitude limited in both directions. Stars brighter than a certain magnitude, in this case about 7·0, cannot be measured satisfactorily. Thus the star *Leiden* 8418 (mag. 6·8) gives very bad results, while the star *Cambridge* 11715 = 52 *Cygni* (mag. 4·4) cannot be measured at all. For such stars the characteristic form of image is marred by photographic diffusion, by diffraction arising from the form of mounting of the plate carrier, and possibly in some cases by the nebulous character of the stars themselves. How far

* The mean of two measures at Oxford has been found to have a probable error of $0''.17$ (*Monthly Notices*, vol. lvii. p. 628). This is independent of errors in the position of an image.

consistency can be secured in measuring fainter stars remains to be seen.

8. The measures already considered are those of stars whose catalogue positions are known, and which are therefore available for an independent reduction of Plate A. By means of them solutions have been obtained for the reduction of the plate according to either of the two methods of measurement employed. We are now in a position to extend the investigation in two directions. In the first place we may inquire in what way the errors of measured coordinates depend on the positions of the stars on the plate, and seek indications of possible distortion of a kind not compensated by the linear formulæ of reduction in which, it is important to remark, no attempt has been made to impose relations on the six constants. In the second place it is desirable to examine, if possible, any influence which the magnitude of a star may have on its estimated position. For the first object in view 45 stars were selected from those measured on Plate B by Mr. Mullis. They consist of nine fairly compact groups, each containing five stars, and situated roughly at the middle points of the sides, at the corners, and at the centre of the plate. The stars are fainter than those previously considered, and probably lie between the magnitudes 9·5 and 10·3. Series of measures were made (1) by myself, according to the method described in § 5; (2) by myself after an interval of some months by the same method, but with the plate placed in orientations 90° and 270°; (3) by Mr. Gray, who used the method of bisection as in the former series. Residuals were formed for each star by applying the solution of §4 to the series (1) and (2), and the solution of § 6 to series (3). In the following table are given the means for each group under the respective numbers. The mean coordinates given in the first two columns are derived from measures of the first series. The means of discordances appended to the table have been obtained by adding the individual residuals, taken positively, and dividing the sum by 45, the number of the stars.

Table II.

		(1)		(2)		(3)	
x_1.	y_1.	δx.	δy.	δx.	δy.	δx.	δy.
4·024	5·138	+1″·8	+0″·5	−0″·4	−1″·9	+1″·0	−0″·3
12·368	5·090	+0·2	−1·4	−0·9	−2·4	−0·2	−0·2
20·499	5·223	−0·7	+0·4	+0·9	−1·6	−0·2	+0·5
4·102	12·576	+0·6	+0·9	−1·8	−0·2	+1·5	−0·1
12·469	12·062	−0·1	−0·1	−0·1	−0·1	+0·6	−0·2
20·081	12·599	+0·8	+0·2	+1·3	−0·6	−0·8	+0·2
4·416	20·026	+0·7	+0·2	−2·4	+1·8	+1·0	+0·2
12·644	19·902	−1·4	+1·0	−1·7	+2·3	−0·8	+0·2
21·216	19·366	−1·5	−0·9	+1·8	−0·1	−1·2	−0·3
Means of discordances without regard to sign		0·85	0·62	1·25	1·23	0·91	0·52

9. The "means of discordances" must be diminished by one-sixth if the general degree of accuracy is to be represented by probable errors. In this way it may be said that the probable error in x and y in series (1) and (3) is about $0''{\cdot}6$, a value previously found, though in both series the y-coordinates seem to be better determined than the x-coordinates. But in the second series of measures, though made with no less care than the first, the residuals are much worse. It is quite clear that the point aimed at in setting on an image was quite different in the two cases, and it seems to me impossible to secure uniformity. Hence the simpler method of bisection appears at present distinctly preferable. The mean group-residuals are in many cases so large that, although the general accuracy is poor, they indicate systematic and not accidental deviations. But no weight can be attached to the amount of such deviations, since the apparent absence of any connexion between the corresponding numbers in the first two series shows that they cannot be reproduced, being dependent on the temporary impression of the observer. In the third series the case may be different. The group-residuals in y are not so large that they need be regarded as otherwise than accidental. The x-residuals, on the other hand, seem to demand a change in the scale value of the abscissæ. This appears from the fact that the means of δx for small, intermediate, and large values of x are $+1''{\cdot}17$, $-0''{\cdot}13$, and $-0''{\cdot}73$ respectively.

10. The stars whose measures have just been discussed lie, with the exception of one group, very near the edges of the plate, the area covered by which is about 2° square. Undoubtedly this increases the difficulty of making measures very considerably. In order to see whether images situated nearer the centre of the field and measured and reduced as before gave better results four additional groups were added. The results are shown in a form similar to the previous table.

Table III.

		(1)		(2)	
x_1	y_1	δx.	δy.	δx.	δy.
		$''$	$''$	$''$	$''$
8·383	8·894	−1·0	−1·2	−0·2	+1·0
16·630	8·635	+0·2	−1·7	0·0	−0·3
8·271	16·650	−1·1	+0·6	0·0	+0·8
17·392	16·138	+2·2	+0·3	−0·3	+0·2
Means of discordances without regard to sign :		1·20	1·00	0·64	0·82

Here the figures under (2) are comparable with those under (3) in the previous table, and show that the accuracy of bisection measures is of much the same order, whether the images are close to the edges or nearer the centre of the plate. The measures used in obtaining the figures of the first four columns above were made in the same manner as series (1) of the preceding paragraph, but at about the same date as series (2) of the same

paragraph. The increased discordances are doubtless due to the fact that after the lapse of time the measures on which they depend are inconsistent with those on which was based the solution employed in the reduction.

11. The question as to how far accuracy depends on the magnitude of the stars whose positions are measured must now be considered, though the disproportion between the times of exposure of the two plates precludes a completely satisfactory answer. A selection of 45 of the faintest stars on Plate B was made, the distribution in nine groups of five stars each being similar to that employed in forming Table II. Measures were again made (1) by myself, (2) by Mr. Gray. The mean residuals are given under corresponding headings in Table IV., the first two columns of which contain the mean coordinates derived from my measures. The 'means of discordances' when compared with the corresponding figures under (1) and (3) in Table II. show that the accuracy attained is still less, as might indeed have been anticipated, than in the case of brighter stars. More especially is this the case with the x-measures obtained by the bisection method. Here again, as was noticed before, the measures seem to demand a different scale value, the mean residuals for the three distinct values of x being $+1''{\cdot}53$, $-0''{\cdot}33$ and $-1''{\cdot}90$. The mean magnitude of the stars probably approaches 11·0. It is a little singular that the decreasing scale value with increasing faintness of the stars is a marked feature in the abscissæ but not in the ordinates. It has already been remarked (§ 9) that little weight can be attached to the residuals obtained by the method of measurement which I adopted. But in this case, if the displacements indicated by the residuals are plotted in a single diagram, they seem to suggest something of the nature of radial distortion. Here the measures might be better suited by *increased* scale values, a fact possibly due to an unconscious effort on the part of the observer to avoid the opposite effect.

TABLE IV.

		(1)		(2)	
x	y	δx.	δy.	δx.	δy.
3·770	5·454	$-0''{\cdot}6$	$-0''{\cdot}7$	$+1''{\cdot}3$	$+1''{\cdot}0$
11·892	4·976	−0·3	−1·3	−0·6	+1·4
20·569	4·972	+0·2	−2·2	−1·0	+0·7
4·071	11·707	0·0	+0·2	+2·0	+0·4
11·716	11·804	−1·0	−0·8	+0·2	−1·0
20·306	11·842	+1·1	−1·2	−1·8	+0·6
4·162	19·970	−1·1	+0·2	+1·3	−1·1
11·996	19·625	−1·4	+1·3	−0·6	+0·4
20·404	19·126	−0·7	−0·2	−2·9	−0·6
Means of discordances without regard to sign:		0·93	1·08	1·57	0·90

12. This note is obviously defective in two respects. The single plate used in the discussion is a long exposure photograph. Hence any deductions which have been made cannot be applied to a reflector photograph taken with a short exposure. But since plates of the latter description are among the material available for the *Eros* parallax determination, they will shortly be exposed to the most searching practical test, and the question of the accuracy of which they are capable must soon find an answer in this way. In the second place the shortness of the exposure of the refractor photograph has made it impossible to push the comparison beyond a certain range of star magnitude which falls short of the limit of the reflector plate. In particular it is not possible to assign the degree of accuracy with which the positions of nebular condensations can be measured. That this is a question of extreme importance is shown in the case of the nebula surrounding Nova *Persei*. But it may well be that it is not susceptible of a direct answer because the mere accuracy of measurement is complicated by the difficulty of choosing a definite point to measure. An illustration is afforded by the attempt to make such measures on the Nova *Persei* nebula at the Lick Observatory.* In this case the difficulty was so great that it was met by placing an artificial mark on the under side of the plate before the photograph was placed in the measuring machine. The fact that it may be necessary to obtain the desired data by measuring such a mark shows that the mere accuracy of measurement is of secondary importance.

13. When full allowance is made for limitations such as have been alluded to in the preceding paragraph the present note still contains results which may probably be considered typical of that large and important class of photographs which have been taken with long exposures by the aid of the reflector of short focal length. Among these results the following points may be summarised :—

(1) The positions of stars with magnitudes between 7·0 and 10·0 can be measured with a probable error of about 0″·6 in both coordinates. As compared with astrographic plates this degree of accuracy is very low and makes it impossible to trace with certainty, from a limited amount of material, anything of the nature of genuine systematic error. On the other hand, the accuracy is of about the same order as that of the A.G. catalogues (§§ 7 and 9).

(2) Measures made on images near the edges of the plate (say 1° from the centre of the field) are certainly not more inaccurate than those made on images lying about 30′ from the centre (§§ 8–10 and Tables II. and III.).

(3) As the magnitude of stars approaches 11·0 the order of accuracy is fairly maintained, but the measures point to scale

* Lick Bulletin, No. 23.

values different from those determined from the brighter stars (§ 11 and Table IV.).

(4) Results are given in Tables I.–IV. for two distinct systems of measurement. Up to a certain point the two methods do not differ much in accuracy. The defect of the first (described in § 5) lies in the uncertainty with which measures can be reproduced after an interval or under different circumstances (§ 9 and Table II). The simple bisection method is much easier, but has the defect that it indicates a change of scale value with star magnitude, this anomaly being conspicuous in the abscissæ, but not in the ordinates (§ 11).

University Observatory, Oxford:
1902 *Nov.* 8.

William Herschel's observed Nebulous Regions, 52 in number, compared with Isaac Roberts' Photographs of the same Regions, taken simultaneously with the 20-inch reflector and the 5-inch Cooke lens. By Isaac Roberts, D.Sc., F.R.S.

William Herschel communicated a paper to the Royal Society which was published in the *Philosophical Transactions* in the year 1811, vol. 74, pp. 269 to 336, under the title of "The Construction of the Heavens"; and in a part of it is a section with the title: "Extensive Diffused Nebulosity" and "Observations of Nebulosities that have not been published before." Fifty-two regions in the sky are stated to be thus affected, and their coordinates of position and the approximate extent of the nebulosity are given in tabular form.

A century nearly has elapsed since these important statements were made, and astronomers still theorise and speculate concerning these nebulous regions in reliance upon the deservedly high eminence and conscientious care with which Herschel executed all his work. The aggregate extent of the diffused nebulosity in these fifty-two regions he estimated to cover about 151·7 square degrees, and he suggested that these represented only a small portion of the total areas in the sky that are similarly affected with nebulosity.

It occurred to me, several years ago, that it would be useful scientific work to photograph, with my 20-inch reflector, the fifty-two regions of diffused nebulosity which had been recorded by Herschel, and thus either verify, or correct, his observations in case of the absence of nebulosity. The photographs would also furnish data, that would be free from human bias and error, in the settlement of the questions, or inferences, which have been founded upon Herschel's records of these nebulous regions.

The work was commenced by me in 1896 and was completed

in 1902. The photographs have been taken, in duplicate, with the 20-inch reflector and a special lens of 5-inch aperture, made by Cooke & Sons, of York, known as Taylor's patent triplet lens. The exposures of the plate with the two instruments were made, in all cases, simultaneously and during equal times; the plates were selected and tested to be practically equal in sensitiveness; the duration of each dual exposure was ninety minutes, which my long previous experience in photographing the stars and nebulæ enabled me to judge that nebulosity of the faintness of stars of 16th to 17th magnitude would leave images upon the photographic plates exposed in the 20-inch reflector, and images of the 14th to 15th magnitude on those exposed in the 5-inch lens camera.

The results which I expected to obtain by these methods were that stars and nebulosity of at least the degree of faintness seen by Herschel would be shown on the reflector plates, and be, therefore, evidence that would be accepted with full confidence by astronomers.

The lapse of so many years since the publication of Herschel's paper, and the consequent difficulty in obtaining copies of it for study, suggested to me the desirability of quoting in my present communication to this Society, so much of it as would explain the view he then entertained concerning these nebulous regions as initial parts in "The Construction of the Heavens." The tabular method adopted by Herschel in publishing the results of his telescopic observations enables me to give the photographic results in a concise and intelligible form, coinciding line by line with his, by comparing the headings and reading the descriptive matter attached to each object respectively.

I shall now quote from Herschel's paper in the *Philosophical Transactions* above referred to.

1. "Extensive diffused Nebulosity.

2. "Observations of Nebulosities that have not been published before.

"It may be easily supposed that in my sweeps in the heavens I was not inattentive to extensive diffusions of nebulosity, which occasionally fell under my observation. They can only be seen when the air is perfectly clear, and when the observer has been in the dark long enough for the eye to recover from the impression of having been in the light.

"I have collected fifty-two such observations in a table, and have arranged them in the order of right ascension; in the first column they are numbered; in the second and third columns are the right ascension and north polar distance of a place, which is the central point of a parallelogram comprehending the space which the nebulosity was observed to fill. They are calculated for the year 1800.

"The length and breadth of the parallelograms are set down in the fourth and fifth columns in degrees and minutes of a great circle, the time taken up in the transit of each parallelo-

gram having been properly reduced to space by the polar distance given in the third column, in order to make it agree with the space contained in the breadth of the zone described by the telescope. The dimensions of the former space, therefore, is in the parallel, and that of the latter in the meridian. My field of view being fifteen minutes in diameter, its extent has been properly considered in the assigned dimensions of the parallelograms. It is, however, evident that the limits of the sweeping zone leave the extent of the nebulosity in the meridian unascertained. The beginning of it is equally uncertain, since the nebulous state of the heavens could only be noticed when its appearance became remarkable enough to attract attention. The ending is always left undetermined; for, as the right ascension was only taken once, I have allowed but a single minute of time for the extent of the nebulosity in that direction, except where the time was repeatedly taken with a view to ascertain how far it went in the parallel; or when the circumstances of its brightness pointed out a longer duration.

"The sixth column of the table contains the size of the observed nebulosity reduced to square degrees and decimals, computed from the two preceding columns; and in the last I have given the account of these nebulosities as recorded in my sweeps at the time they were made, namely, within a period of nineteen years, beginning in 1783 and ending in 1802.

"When this account says 'Affected,' it is intended to mean that the ground upon which, or through which, we see or may see stars is affected with nebulosity.

"In looking over this table it may be noticed that I have inserted several nebulosities that were only suspected. Had I been less scrupulous at the time of observation the word suspected would generally have been omitted; for with this nebulosity, as well as with the great number of nebulæ that in my catalogues are marked suspected, I have, almost without exception, found in a second review that the entertained suspicion was either fully confirmed, or that, without having had any previous notice of the former observation, the same suspicion was renewed when I came to the same place again.

"When these observations are examined with a view to improve our knowledge of the construction of the heavens, we see in the first place that extensive diffused nebulosity is exceedingly great indeed; for the account of it, as stated in the table, is 151·7 square degrees; but this, it must be remembered, gives us by no means the real limits of it, neither in the parallel nor in the meridian; moreover, the dimensions in the table give only its superficial extent; the depth or third dimension of it may be far beyond the reach of our telescopes; and when these considerations together are added to what has been said in the foregoing article, it will be evident that the abundance of nebulous matter diffused through such an expansion of the heavens must exceed all imagination.

"By nebulous matter I mean to denote that substance, or rather those substances which give out light, whatsoever may be their nature, or of whatever different powers they may be possessed.

"Another remark of equal importance arises from the consideration of the observed nebulosities. By the account of the table we find that extreme faintness is predominant in most of them ; which renders it probable that our best instruments will not reach so far into the profundity of space as to see more distant diffusions of it. In No. 44 of the table we have an instance of faint milky nebulosity, which, though pretty bright in some places, was completely lost from faintness in others ; and No. 46 confirms the same remark. It has also been mentioned in the first article that the nebulosity in V. 14 was brighter in three or four places than in the rest. The stars also of the Milky Way which were scattered over it, and were generally very small, appeared with a brilliancy that will admit of no comparison with the dimness of the brightest nebulosity. In consequence of this we may already surmise that the range of the visibility of the nebulous matter is confined to very moderate limits."

The photographs were taken with an exposure of the plates during 90^m in duplicate simultaneously with the 20-inch reflector and with the 5-inch Cooke lens. In all cases the objects were photographed when on or near the meridian. The reflector plates measure 2° by 2°, and the stars upon them are shown to the faintness of the 16th or 17th magnitude. On the 5 inch camera plates they are shown to the faintness of about the 14th or 15th magnitude.

Herschel's Nos.	R.A. 1900. h m s	Decl. 1900. ° ′	Herschel's descriptions in the Phil. Trans. 1811.	Dates when photographs were taken.	Isaac Roberts's descriptions of his photographs.
1	0 10 8	9 26	Much affected with nebulosity.	1900 Nov. 22	Sky clear, stars small and faint and few in number ; large areas void of stars ; no nebulosity on plate.
2	0 17 37	3 59	Much affected.	1899 Sept. 5	Sky clear ; stars small and faint and not very numerous ; large areas void of stars ; no nebulosity on plate ; film dark.
3	0 22 23	29 9	Affected.	1899 Sept. 9	Sky clear ; stars small and very, very numerous, one star of 5·9 mag. = D.M. 75, zone 28°, on plate ; small areas void of stars ; no nebulosity.
4	0 25 37	3 59	Much affected.	1900 Nov. 22	Sky clear ; stars few and faint, large areas void of stars ; no nebulosity on plate.
5	0 30 11	23 25	Much affected.	1900 Oct. 27	Sky clear ; stars faint and numerous ; nebulæ H III. 476 and N.G.C. 169, d'Arrest and Ld. R. together with other fainter ones on plate ; many areas void of stars ; no diffused nebulosity.

Herschel's Nos.	R.A. 1900. h m s	Decl. 1900. ° ′	Herschel's descriptions in the Phil. Trans. 1811.	Dates when photographs were taken.	Isaac Roberts's descriptions of his photographs.
6	0 36 28	0 29	Appeared to be affected with very faint nebulosity.	1899 Oct. 28	Sky very clear; stars small and very few in number; large areas void of stars; some small nebulæ on plate; no diffused nebulosity.
7	0 38 0	41 10	Affected with nebulosity.	1895 Oct. 17	Sky very clear; stars crowded on plate; many small areas void of stars; several photographs have been taken of this region, which includes the great Andromeda nebula M 31, part of the *n.f.* end of which would cross Herschel's field of view in this sweep.
8	0 39 27	39 16	Unequally affected.	1900 Oct. 17	Sky clear; stars crowded on plate; many small areas void of stars; part of *s.p.* end of M 31 on plate; no other diffused nebulosity.
9	0 41 19	43 30	Suspected faint nebulosity.	1900 Oct. 26	Sky clear; stars small and crowded on plate; many small areas void of stars; no diffused nebulosity.
10	0 48 38	43 35	Suspected faint nebulosity.	1900 Oct. 26	Sky clear; stars small and crowded on plate; numerous areas void of stars; nebula N.G.C. 317 on plate; no diffused nebulosity.
11	1 41 8	29 48	Suspected to be tinged with milky nebulosity.	1900 Nov. 27	Sky clear; stars small and numerous; large areas void of stars; no nebulosity.
12	2 27 55	19 0	Much affected with nebulosity.	1900 Dec. 13	Sky clear; stars small and not very numerous; large areas void of stars; some very small and faint nebulæ on plate; no diffused nebulosity.
13	4 2 14	25 11	Much affected.	1901 Feb. 13	Sky very clear; stars small and numerous; large areas void of stars; no nebulosity.
14	4 23 51	35 7	Suspected pretty strong nebulosity.	1901 Feb. 13	Sky very clear; stars small and crowded on *s.* and *s.p.* sides, but few on the rest of the plate; large areas void of stars; nebula H I. 217, and also a 10th mag. star surrounded by very faint nebulosity, 11′·5 *n.f.* H I. 217 on plate; no nebulous region.
15	4 24 51	35 8	Suspected nebulosity.		
16	4 26 29	−7 30	Strong milky nebulosity.	1901 Feb. 14	Sky clear; stars small and very few on plate; large areas void of stars; no nebulosity.

Herschel's Nos.	R.A. 1900. h	m	s	Decl. 1900. °	′	Herschel's descriptions in the Phil. Trans. 1811.	Dates when photographs were taken.	Isaac Roberts's descriptions of his photographs.
17	4	29	2	20	50	Much affected.	1901 Feb. 15	Sky very clear; stars small and very numerous; small areas void of stars; no nebulosity.
18	4	44	5	20	50	Much affected.	1901 Feb. 15	Sky very clear; stars small and crowded on plate; small areas void of stars; no nebulosity.
19	4	52	17	26	45	Strong suspicion of very faint milky nebulosity.	1901 Mar. 9	Sky clear; stars small and very few; large areas void of stars; no nebulosity.
20	5	15	50	25	1	Very much affected.	1901 Mar. 12	Sky clear; stars small and very few on plate; large areas void of stars; no nebulosity.
21	5	19	20	25	1	Affected.		
22	5	28	53	−6	56	Affected with milky nebulosity.	1901 Mar. 13	Sky clear; stars not very numerous; large areas void of stars; H IV. 33 Orionis on plate; no nebulosity.
23	5	30	10	−2	43	Affected.	1901 Mar. 12	Sky clear; stars small and very few; large areas void of stars; no nebulosity.
24	5	31	56	−4	18	Visible and unequally bright nebulosity. I am pretty sure that this joins to the great nebula in Orion.	1902 Mar. 5	Sky very clear; stars small and not very numerous; areas void of stars; no nebulosity on plate.
25	5	35	34	−2	31	Diffused milky nebulosity.	1900 Jan. 25	Sky very clear; stars very numerous on *p.* half of plate, but few on the *f.* half, where there are large areas void of stars: large cloud of nebulosity *n.f.* ζ Orionis with broad division void of stars, but with some nebulosity in *s.f.* to *n.p.* direction; other divisions break up the cloud into separate masses. To the *s.* of ζ is a stream of nebulosity 54 minutes of arc in length, with an embayment free from nebulosity dividing it in halves. Another faint nebulosity extends from ζ 27 minutes of arc towards the *s*, *s.p.* and *n.p.* The star D.M. 1001, zone −1°, is in the midst of nebulosity, and it has a companion on the *s.p.* side. The star D.M. 1005, zone −1°, is involved in a large cloud of streaky nebulosity and it has a companion on *p.* side. The star D.M. 1345, zone −2°, is H IV. 24, N.G.C. 2023. It is in the midst of a large dense streaky cloud of nebulosity which has in it condensations and remarkable rifts free from nebulosity; near the *s.* end of one of these rifts is a 12th mag. star. The star D.M. 1350, zone −2°, is in the midst of a cloud of nebulosity with some faint structure in it; it has a faint companion on the *n.p.* side. The region here referred to, which covers 4 square degrees of the sky, has so many remarkable features that it is necessary, in order to make it intelligible to the reader, to present the photograph annexed along with the above description.

Herschel's Nos.	R.A. 1900. h m s	Decl. 1900. ° ′	Herschel's descriptions in the Phil. Trans. 1811.	Dates when photographs were taken.	Isaac Roberts's descriptions of his photographs.
26	5 36 52	−6 57	A pretty strong suspicion of nebulosity.	1901 Mar. 22	Sky clear; stars small and few; large areas void of stars; no nebulosity.
27	5 43 11	+1 8	Affected with milky nebulosity.	1901 Mar. 13	Sky clear; stars very few in number; large areas void of stars; no nebulosity.
28	6 1 1	+3 44	Much affected.	1902 Jan. 29	Sky clear; stars crowded on *n.f.* and *s.p.* sides; areas void of stars; no nebulosity.
29	6 0 54	−20 27	Affected.	1902 Mar. 6	Sky clear; stars small and very numerous; many areas void of stars; no nebulosity.
30	6 40 7	41 16	Affected.	1901 Mar. 22	Sky clear; stars few in number; large areas void of stars; cluster H VIII. 71 on plate; no nebulosity.
31	9 27 32	−18 27	Affected.	1902 Mar. 6	Sky clear; stars small and few in number; large areas void of stars; no nebulosity.
32	9 36 43	71 13	Much affected with very faint whitish nebulosity.	1901 Apr. 12	Sky clear; stars small and numerous; several large areas void of stars; no nebulosity.
33	10 11 50	−9 3	Very faint whitish nebulosity.	1901 Apr. 15	Sky clear; stars small and numerous; large areas void of stars; no nebulosity.
34	10 22 25	51 32	Much affected.	1901 Apr. 13	Sky clear; stars small and not numerous; large areas void of stars; no nebulosity.
35	10 40 59	62 45	Affected with very faint nebulosity.	1901 Apr. 14	Sky clear; stars small and not very numerous; large areas void of stars; no nebulosity.
36	11 4 30	62 44	Affected.	1901 Apr. 15	Sky clear; stars small and numerous; areas void of stars; several small faint nebulæ on plate; no diffused nebulosity.
37	12 2 5	30 37	Affected with whitish nebulosity.	1901 Apr. 17	Sky clear; stars small and few in number; large areas void of stars; H II. 321 and H II. 802 on plate; no nebulosity.
38	12 12 40	30 37	Affected with whitish nebulosity.	1901 Apr. 18	Sky clear; stars few in number; large areas void of stars; four small prominent nebulæ on plate; no diffused nebulosity.
39	13 12 15	34 8	Much affected.	1901 Apr. 17	Sky clear; stars not very numerous; large areas void of stars; no nebulosity.

S

N

PHOTOGRAPH OF NEBULOSITY ROUND ζ ORIONIS

BY ISAAC ROBERTS, D.Sc., F.R.S

S

N

PHOTOGRAPH OF NEBULOSITY HERSCHEL V. 37 CYGNI

By Isaac Roberts, D.Sc., F.R.S.

Herschel's Nos.	R.A. 1900. h	m	s	Decl. 1900. °	′	Herschel's descriptions in the Phil. Trans. 1811.	Dates when photographs were taken.	Isaac Roberts's descriptions of his photographs.
40	14	2	20	34	8	Very much affected, and many faint nebulæ suspected.	1899 June 2	Sky clear; stars small and not numerous; areas void of stars; no nebulosity.
41	15	9	37	18	57	Affected with very faint nebulosity	1899 June 12	Sky clear; stars small and not very numerous; areas void of stars; no nebulosity.
42	21	3	26	−1	53	Much affected with whitish nebulosity.	1902 Nov. 4	Sky clear; stars very numerous; no nebulosity. Herschel's sweep 42, as given in the *Phil. Trans.* (R.A. 1800 = $20^h\ 58^m\ 20^s$, N.P.D. 1800 = 92° 17′) is not in sequence; as this may be due to a typographical error in one of the coordinates, a plate corresponding to (R.A. 1800 = $20^h\ 38^m\ 20^s$, N.P.D. 1800 = 92° 17′) was taken on 1897 Aug. 28 as follows:
							1897 Aug. 28	Sky very clear; stars crowded on plate; no nebulosity.
43	20	53	15	16	44	A good deal affected.	1897 Oct. 20	Sky clear; stars crowded on plate; no nebulosity.
44	20	54	34	43	32	Faint milky nebulosity scattered over this space, in some places pretty bright.	1896 Oct. 10	Sky very clear; stars crowded on parts of plate; large areas void of stars on others; nebula IJ.V.37, N.G.C. 7000, forms part of this region; the photograph shows it as a magnificent object. I have published a photograph of this region in Vol. II. of *Stars, Star-Clusters and Nebulæ*. Pl. 24, p. 155, and also in *Knowledge*, 1898 Nov. 1. A copy is also annexed to this paper.
45	20	57	34	−1	34	Much affected with whitish nebulosity.	1897 Sept. 21	Sky clear; stars small and numerous; no nebulosity.
46	20	56	55	43	16	Suspected nebulosity joining to plainly visible diffused nebulosity.	1896 Oct. 10	Regions 44 and 46 are on the same plate; see description given above, No. 44.
47	21	5	8	14	21	Affected.	1899 Aug. 6	Sky clear; stars small and crowded on plate; no nebulosity.
48	21	34	15	10	19	Much affected.	1898 Oct. 12	Sky clear; stars small and numerous; areas void of stars; no nebulosity.

Herschel's Nos.	R.A. 1900. h m s	Decl. 1900. ° ′	Herschel's descriptions in the Phil. Trans. 1811.	Dates when photographs were taken.	Isaac Roberts's descriptions of his photographs.
49	21 46 52	21 31	Affected.	1899 Aug. 9	Sky clear; stars small and crowded; areas void of stars; no nebulosity.
50	22 57 24	25 45	Much affected.	1898 Sept. 20	Sky clear; stars very numerous; areas void of stars; no nebulosity.
51	22 57 54	25 45	Affected.		
52	23 0 17	29 17	A little affected.	1900 Oct. 27	Sky clear; stars small and very numerous; areas void of stars; H II. 212 on plate; no diffused nebulosity.

Conclusions.

The final results of the correlation of Herschel's fifty-two nebulous regions and my photographs can be given in a few words, as follows:

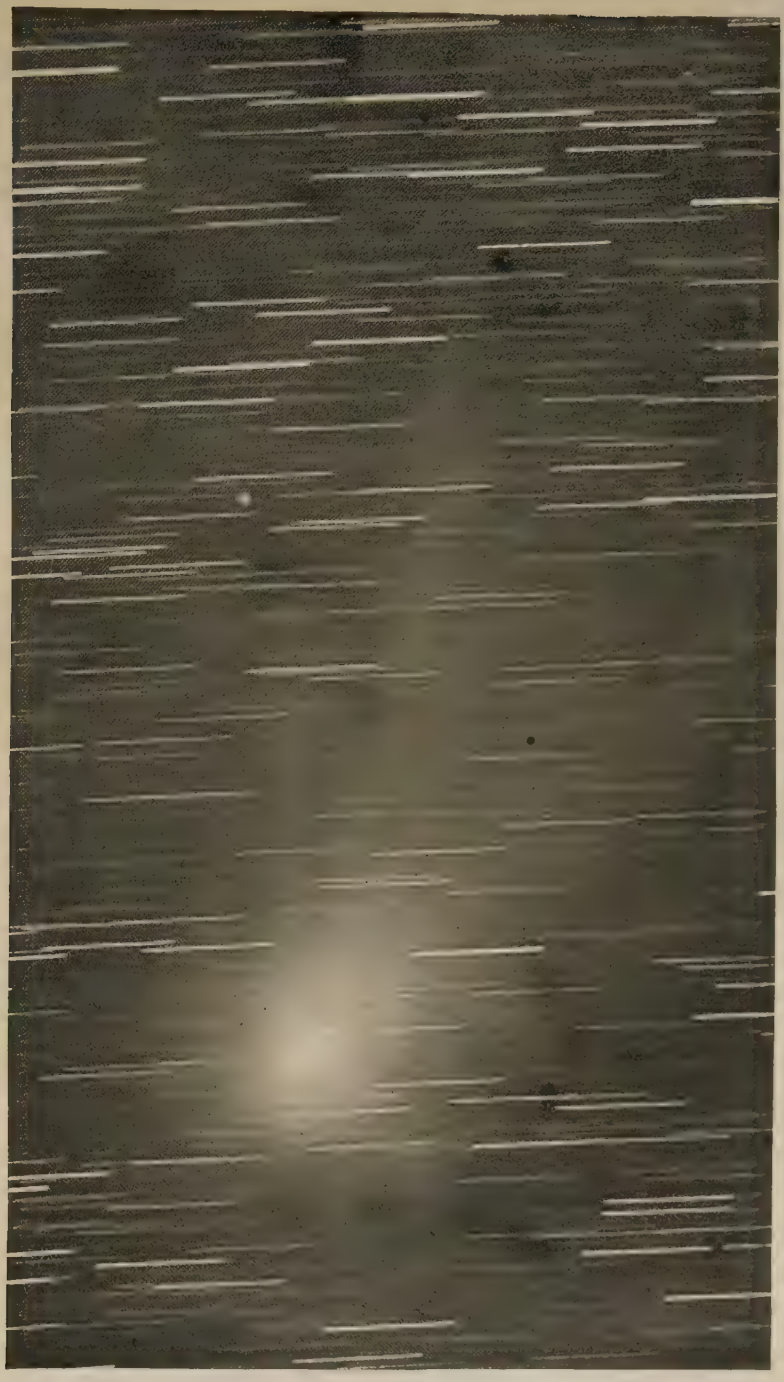
Of the fifty-two nebulous regions described by Herschel, the photographs show diffused nebulosity on four of them only; there is no visible trace of diffused nebulosity on forty-eight of the areas, but on the remaining four, which are Nos. 7, 25, 44, and 46 respectively in the table, there is nebulosity with remarkable characteristic features, and these are delineated upon three of the photographs, regions Nos. 44 and 46 being on one plate

Two photographs have been enlarged as paper prints, and are reproduced on Plates 1 and 2, an examination of which will convey a more accurate knowledge of the objects than is possible by any descriptive matter.

Note on Photographs of Comet b 1902 (*Perrine*), *taken at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Photographs of Comet *b* 1902 (Perrine) were obtained on twenty-seven nights, between September 6 and October 29. With the exception of one or two photographs at the beginning, the 30-inch reflector was used. Twenty-nine of the photographs of short exposure showing a well-defined nucleus have been selected for measurement for the position of the comet, and the results deduced from them will be communicated to the Society later. The exposures with the 30-inch reflector ranged from 5^m to $\frac{1}{2}^m$, according to the brightness of the comet. Besides these, seven photographs with long exposures, ranging from 13^m to 70^m, were also taken, those obtained on September 26 and September 29 (the latter reproduced on Plate 3) being of special interest. On these days the exposures were 53^m and 62^m



COMET *b* 1902. SEPT. 29

ROYAL OBSERVATORY, GREENWICH (EXP. 62 MIN.)

respectively, and "Lightning" plates were used. The telescope was guided in R.A. and Dec. so as to allow for the motion of the comet, and the stars are consequently shown as trails representing the comet's motion during the exposure. On each of these photographs there is a bright globular head, several short tails and one long faint tail extending a degree from the comet. On the photograph taken on September 29 as many as seven tails of different lengths, divided by six narrow rifts, can be distinguished. The direction of the long tail on both photographs is in approximately the opposite direction to the projection of the Sun on the plate.

Stereoscopic Pictures of Comet Perrine. By Max Wolf, Ph.D.

The two photographs presented have been made from four plates which I took with my 16-inch double photographic telescope. The two 16-inch lenses are so nearly alike in size and focal length that the idea occurred to me to obtain a stereoscopic effect by exposing plates at slightly different times with the two lenses, so that the movement of the Earth and of the comet should give the necessary shift, and I might hope to obtain the right effect by combining two plates. At first I hoped to be able to use for this purpose two successive plates made with the same telescope; but this gave no satisfactory results: the movement of the comet was too quick.

Indeed the first of the two stereos (566 and 567) gives an almost surprising view in a stereoscope. The comet hangs free in space, very near to the observer, and the trails of the fixed stars lie far behind. Seen with the naked eye the nebulous mass of the comet on the photo is flat and opaque; in the stereoscope it looks like a cloud of dust, condensed towards its centre. The trails of the fixed stars visible through it are seen far behind, giving out their own light through the tail.

The dates of these two pictures are:

No. 566: 1902 Sept. 25, M.T. Königstuhl $7^h\ 45^m{\cdot}5$–$8^h\ 45^m{\cdot}5$
No. 567: „ „ „ „ „ $7^h\ 55^m{\cdot}0$–$8^h\ 55^m{\cdot}0$

and the second stereo:

No. 576: 1902 Oct. 1, M.T. Königstuhl $7^h\ 48^m{\cdot}1$–$8^h\ 48^m{\cdot}1$
No. 577: „ „ „ „ „ $8^h\ 3^m{\cdot}1$–$9^h\ 3^m{\cdot}1$

Here the movement of the comet was still a little too great, so that the stereoscopic effect is not as clearly visible for every observer as on the first pair: it requires to be looked at from a great distance. Still, on this picture the tail of the comet

is more interesting. The scale of the four pictures is exactly the same, and shows the increase of size and of the movement of the comet. Indeed, on a picture of October 8 the tail covers the whole plate and is longer than $3\frac{1}{2}$ degrees.

Now the stereoscope allows us to decide upon an interesting problem which occurred to me in photographing comets.

Examining the star trails on a photograph of a comet I found that every trail of a faint fixed star entering the tail of the comet, instead of becoming fainter by absorption, increases sensibly in brightness. It is a very striking appearance, and I sought for an explanation. I thought that it was produced by the increase of rapidity of the photographic film caused by the faint light of the tail, the so-called "Vorbelichtung." Now the stereo seems to show that this is wrong, and that the phenomenon is an optical effect. Viewing one print in the stereoscope with one eye—closing the other—we see the striking brightening of the star trails in the tail; opening the second eye we see this effect disappear, and the light of the dust of the comet's tail become immediately separated from the light of the fixed star, so that the star trail retains nearly exactly its own brightness while travelling behind the tail of the comet.

In making reproductions of original plates of comets I always found the sky very much brighter on the side of the tail, even many degrees from the comet's nucleus and over a very large area, so that there is no doubt that the cometic matter fills all space around.

Heidelberg, Königstuhl Astrophys. Observatory:
1902 October 22.

[The stereoscopic pictures are placed in the Library.]

Note on a Comparison of Groombridge's Catalogue (1810) *with the Greenwich Second Ten-Year Catalogue* (1890), *with reference to the Question of an Apparent Rotation of the Brighter Fixed Stars as a Whole with respect to the Fainter Stars.*

(*Communicated by the Astronomer Royal.*)

In a preliminary note in *Astronomische Nachrichten*, No. 3800, Sir David Gill concludes, as the result of a comparison of Taylor's Madras Catalogue with modern Cape Catalogues, that the brighter stars rotate with respect to the fainter stars as a whole. As in the preparation of the Greenwich Second Ten-Year Catalogue for 1890 Groombridge's Catalogue for 1810 had been brought up to 1890 for comparison, means were to hand to see whether Dr. Gill's conclusions were supported by a comparison of observations made with an interval of eighty years in the part of

the sky from 35° to 50° N.P.D. Nearer the pole the number of stars is too few to make a comparison of much value.

In the following comparison, stars were excluded for which a proper motion is given in the Greenwich Ten-Year Catalogue for 1890. This was done merely as a matter of convenience in diminishing numerical work, as, in view of the re-reduction of Groombridge's observations now in progress, this comparison is only to be regarded as a provisional one. This method of selection leaves out stars of large proper motion, and also all the stars observed by Bradley for which the proper motions obtained by Auwers were adopted in the Ten-Year Catalogue, but as regards cosmical distribution of the stars is quite fortuitous. When the re-reduction of Groombridge's observations is finished a more complete and detailed comparison will be made, but it is of interest to place the present comparison on record in view of Sir David Gill's paper.

Excess of Right Ascension of the Greenwich Second Ten-Year Catalogue (1890) *over Groombridge's Catalogue* (1810) *for the Stars between* 35° *and* 50° *N.P.D.*

Limits of R.A.	Mag. 5·0–5·9.		Mag. 6·0–6·9.		Mag. 7·0–7·9.		Mag. 8·0–8·9.	
	Diff.	No. of Stars.	Diff.	No. of Stars.	Diff.	No. of Stars.	Diff.	No. of Stars
h h	s		s		s		s	
0– 3	+·29	16	+·06	50	+·09	136	+·03	89
3– 6	+·22	16	+·13	41	+·11	94	+·01	67
6– 9	+·23	8	−·06	43	−·06	72	−·12	38
9–12	·00	11	+·04	27	−·14	77	−·04	28
12–15	−·09	8	−·03	22	−·03	39	−·13	23
15–18	−·06	9	+·03	43	−·01	69	+·06	14
18–21	+·22	18	+·13	104	+·12	225	+·05	181
21– 0	+·18	18	+·15	75	+·18	169	+·07	131
Mean ...	+·12	104	+·06	405	+·03	881	−·01	571

It is to be noted that no magnitude-corrections have been applied in forming these results, and although the progressive diminution of the means agrees in sign with the results given by Sir David Gill, it hardly affords sufficient evidence, in view of the unknown magnitude-correction, on which to base any conclusion as to a cosmical movement of the nature indicated by Sir D. Gill.

It will be of interest to see how far these results will be affected by the re-reduction of Groombridge's observations now nearly completed.

Newcomb's Fundamental Catalogue: Notes and Errata.

By W. G. Thackeray.

Notes.

The places and proper motions of the two following stars, the first in R.A. and Dec., the second in R.A. only, do not appear to be correct, apparently owing to the Piazzi places being discordant.

The following data, reduced to 1900 with Struve's precession and no proper motion, have not been corrected for systematic differences of the catalogues.

The Groombridge Catalogue places are from the new reductions.

No. 1464.

Catalogue.	R.A. 1900.	Epoch.	No. of Obs.	Dec. 1900.	Epoch.	No. of Obs.
	h. m. s.			° ′ ″		
Piazzi xxii., 36	22 9 34·44	1800	10	38 43 9·5	1800	7
Groombridge, 3716	34·88	1809·9	6	6·54	1809·9	6
Radcliffe, 5612	34·97	1849·7	2	7·30	1849·8	2
Greenwich (1860), 1859	34·99	1859·7	4	6·79	1859·8	6
Brussels, 6189	34·95	1869·1	3	7·13	1869·1	3
Greenwich (1872), 2085	34·98	1872·2	5	6·76	1872·2	10
„ (1880), 3718	35·03	1881·1	3	7·05	1881·4	5
„ (1890), 6189	35·12	1892·3	5	7·28	1891·1	10
Newcomb, 1464	35·489	1900·0		6·55	1900·0	
	P.M. + 0ˢ·0126			P.M. + 0″·021		

No. 1556.

Catalogue.	R.A. 1900·0.	Epoch.	No. of Obs.
	h. m. s.		
Piazzi xxiii., 101	23 25 23·63	1800	19
Groombridge, 4078	24·50	1809·8	12
Radcliffe, 6092	24·45	1845·1	5
Brussels, 10536	24·60	1868·1	4
Helsingfors, 14105	24·55	1871·8	2
Greenwich (1890), 6681	24·68	1893·3	10½
Newcomb, 1556	24·960	1900·0	
	P.M. + 0ˢ·0072		

Errata.

Newcomb No.

60. Secular variation in Dec. 1900 *for* −44″·18 *read* −54″·18. No. in Bradley Insert 65.

1589. Seconds of R.A. 1875 and 1900 appear to be 1ˢ too great.

Another Form of Micrometer for Measuring Star Positions.

By H. C. Russell, C.M.G., F.R.S.

When working with the beautiful Star-measurer designed by Sir David Gill, I noticed the accuracy of the work, and its convenience for the purpose; but I was struck with the time taken to move the micrometer spider lines across a réseau square, and I came to the conclusion that it would be an improvement to have a quicker motion than the screw provided, without reducing the accuracy. I have also provided that microscopes should conveniently and rapidly read the positions in minutes, seconds, and tenths of seconds of arc, and the reader writes them down with a small type-writer, so that he need not take his eye from the microscopes, and can easily read and write them down in half a minute. So much is claimed; let us see the mechanism.

Fig. I. (Plate 4).—Here the machine is in complete order for use; A is a thin board to protect the graduated circles from the observer's breath, and the thin nickel-plated circles B and C were raised to give more room over the axes of the graduated circles.

Fig. II.—The thin board removed exposes the two graduated circles 5 inches in diameter: they are graduated only $\frac{1}{4}$ of the circumference. This space is divided into 5 minutes, and each minute is subdivided into 60 seconds—that is, into the space of a réseau on the photographic plates. Each microscope has a grating in the eyepiece which divides every second into 10 parts. When at work the lower parts of the observer's hands rest comfortably on B and C′, and the fingers take H H, and turning them either way move the circles, and they are read by the microscopes F and G, and recorded by a small type-writer (not shown in the photograph), so that each observer is conveniently placed, and has comfortable positions for eyes and hands. (Of details later.)

Plate 6 shows the details of the small cast-iron chambers under the graduated circles. Starting from plan and section, J is the exact counterpart of an ordinary microscope with cross motions (except that it has no screws so far), but instead of screws at H H there are placed pieces of the balance-spring of a watch, H, by which, instead of screws, the micrometer is moved. (See X Y plan and section and elevation and section Z Z.) H is attached to C in each axis of the graduated circles by a screw I; and the milled-headed screw J E and G, resting on the other wheels D D, is for regulating the friction

on that wheel ; without it the milled head is too free to feel pleasant when bisecting a star image carefully and accurately.

The adjustment of focus and of the scale—that is, the graduations on the circles—to be exactly equal to a réseau square, five millimetres, is provided for by the compound screws K K K K (*Fig. II.*), and the focal adjustment of the microscope.

The chambers shown in Plate 6 are $\frac{1}{4}$-inch thick, are made of cast-iron, and are *dust proof*, thus keeping the changes of temperature very slow, and the effect as far as possible uniform over all the measuring parts.

Little need be said of the sliding frames for placing a réseau square in its place. There is a large hole as close to the micrometer J and as near that point as possible, so that a spot of the réseau plate under and about the microscope is easily seen for placing any part under the microscopes.

Fig. III. (Plate 5) shows the arrangement of the slides for adjusting the photograph ; one ∧ guide controls the straight-line motion with perfect accuracy, the other support is on friction rollers, and the wire over the pulley carries a balance weight. The motion is all that can be desired ; the cross slide hangs on the upper edge of the larger slide, and works very satisfactorily.

Sydney Observatory: 1902 *July* 29.

A Standard Scale for Telescopic Observations.

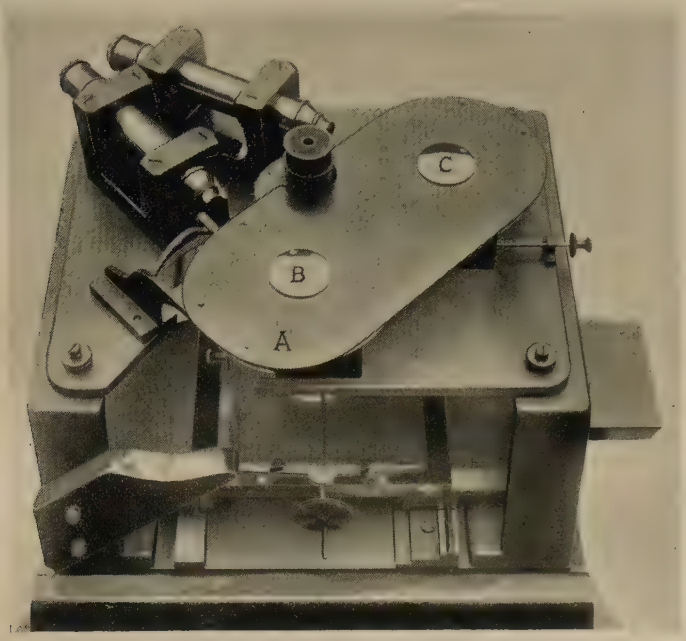
By Percival Lowell.

(*Communicated by the Secretaries.*)

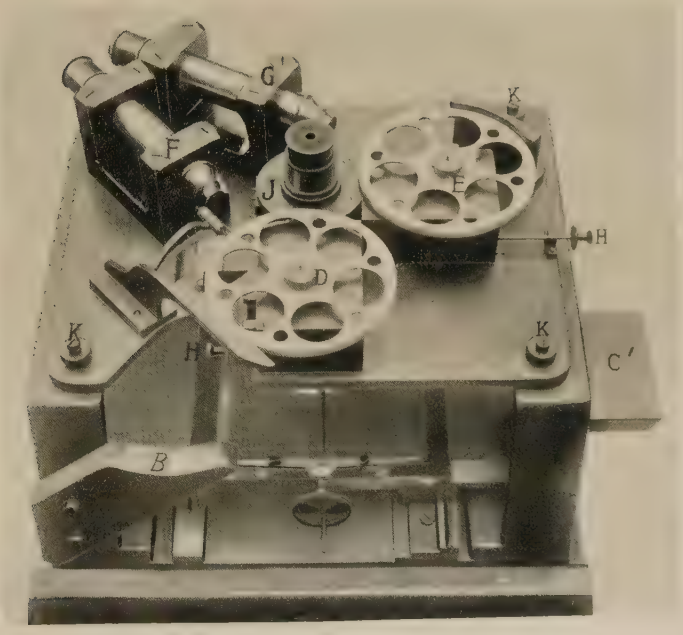
1. *Present State.*—At present there exists no criterion among astronomers for the weight to be attached to any given observation due to the atmospheric conditions under which it is made. Yet these atmospheric conditions are among the most important factors entering into an astronomic observation. They are far more to the point than the size of the instrument. For our telescopes have long since outstripped the conditions under which they are put to work ; the great bar to advance to-day, whether visually, photographically, or spectroscopically, being not instrument but atmosphere. Each man realises this, but marks his own work on his own scale, as if he should take his own foot as the unit of length.

2. *Difficulties of this Condition.*—In consequence no absolute value is assignable to any man's work, and no comparison between different men's work is possible whether in accuracy or credibility. The practical outcome is that the only test is the

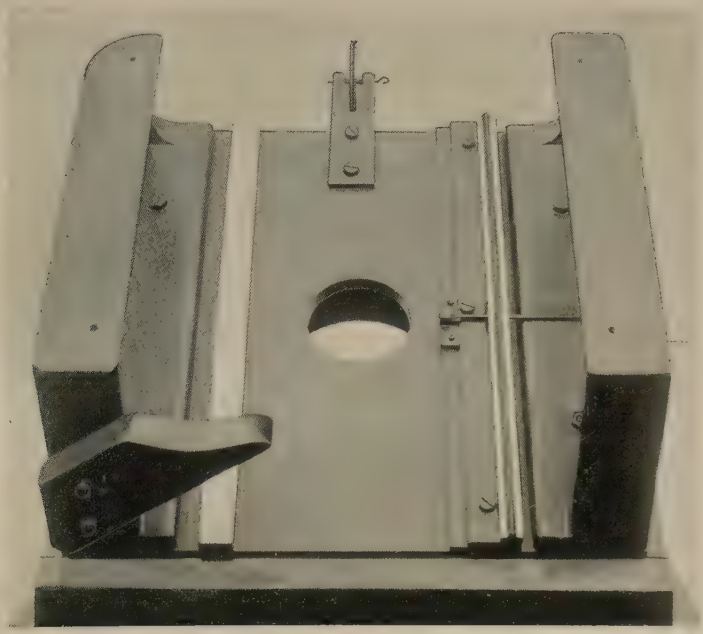
I.



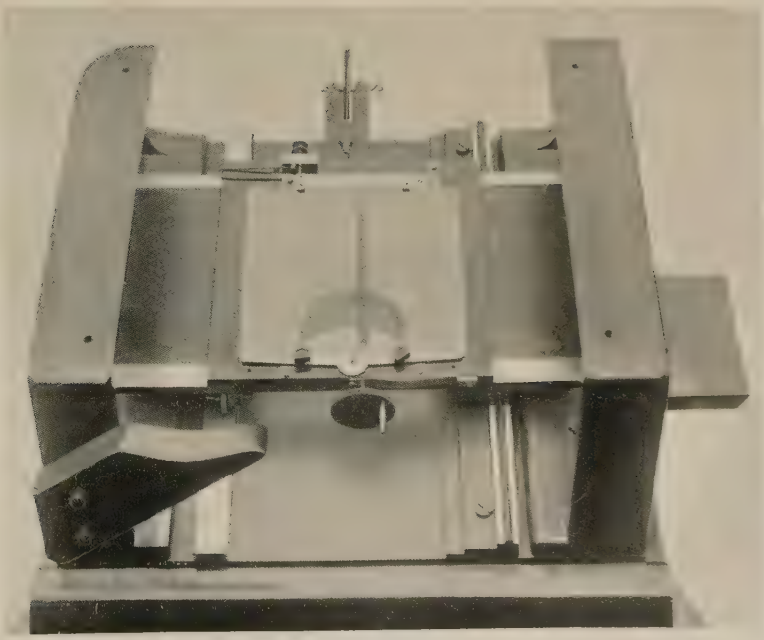
II.



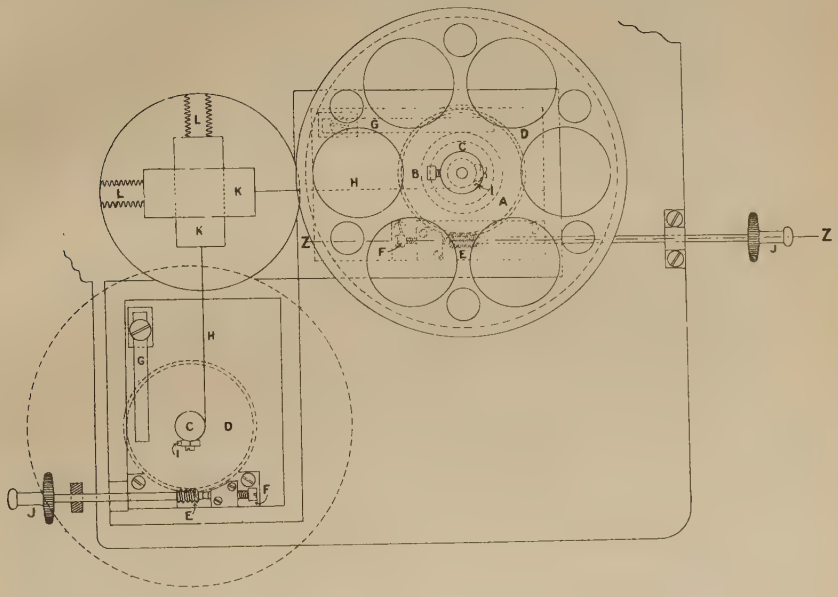
III.



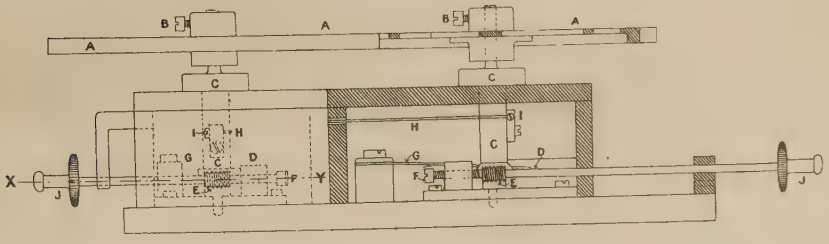
IV.



PLAN AND SECTION ON XY



ELEVATION AND SECTION ON ZZ



test of time, and while the world is waiting for confirmation of any new result just so many years are lost.

As important is the incapacity to leave permanent records of observations capable of being compared with newer ones as time rolls on.

3. *A Change necessary.*—A change in this state of things is imperatively needed. It is time a standard scale for observations were introduced similar to what the metric system is, that it may do what that does for physics generally.

4. *Possibility of a Criterion.*—Until lately such a scale has not been feasible owing to ignorance of the conditions upon which it must be based. Studies, however, directed to that end first at Arequipa and then at Flagstaff during the past few years have resulted in the knowledge of the conditions which constitute good or bad seeing, and have thus enabled an absolute scale to be constructed.

5. *The Criterion.*—The basis of the matter lies in the discovery that systems of waves traverse the air, several of these systems being present at once at various levels above the Earth's surface. The waves composing any given system are constant in size, and differ for the different currents all the way from a fraction of an inch to several feet in length. If the wave be less than the diameter of the object-glass from crest to crest the image is confused by the unequal refraction from the different phases of the wave. If the wave be longer than this a bodily oscillation of the whole image results. The first is fatal to good definition, the second to accurate micrometric measurement.

It is possible to see these waves by taking out the eyepiece and putting one's eye in the focus of the instrument when the tube is pointed at some sufficiently bright light. It is further possible to measure their effect by careful noting of the character of the spurious disc and rings made by a star and the extent of the swing of the image in the field of view. By combining the amount of confusion with the degree of bodily motion of the resulting image the definition at any time and place can be accurately and absolutely recorded.

The increasing perfection of the optical image of a star testifies to the increasing lack of damaging currents with reference to the object-glass used. It records all the waves below a certain wave-length. Similarly the amount of bodily motion registers all those above that length. The two taken together give account of all the currents independent of the glass.

6. *The Scale.*—It is therefore necessary only to agree upon some size of glass for making the fundamental tests and then to reduce the results to any aperture by relations which will be found set forth in a pamphlet by Mr. Douglas made at this observatory, entitled "Scales of Seeing."

The most feasible size for comparison purposes seems the 6-inch aperture.

The scale it is proposed to adopt is therefore as follows :—

With a 6-inch glass—

0 Disc and rings confused and enlarged.
2 Disc and rings confused but not enlarged.
4 Disc defined ; no evidence of rings.
6 Disc defined ; rings broken but traceable.
8 Disc defined ; rings complete but moving.
10 Disc defined ; rings motionless.

Synchronous determination of the amount of bodily motion of image in seconds of arc.

Expedition for the Ascertaining of the best Location of Observatories. By Percival Lowell.

(*Communicated by the Secretaries.*)

In order to discover the best place or places for the location of telescopes in the future it is proposed to send observers furnished with similar instruments and identical instructions to all promising parts of the Earth's surface.

Two desert belts girdle the Earth in the sub-tropical regions of *Capricorn* and *Cancer*, and from the meteorologic conditions there prevailing these belts offer the greatest promise to the astronomer. In the northern hemisphere the belt shows itself first in the Sahara of Africa, then in Arabia, then in the desert of Gobi, and crossing the Pacific crops out again in Arizona and Mexico. Of these the two with the greatest height for their plateaux are Arizona and Mexico and the desert of Gobi. In the southern hemisphere we have the veldt of southern Africa, the western part of Australia, and finally the west coast of Peru and Bolivia. Of these the last is the highest and the Transvaal the next.

With regard to these places we have the most systematic series of records from Arizona, the next so from Peru, some slight knowledge of the Sahara, and next to none of any other locality.

Although the desert belts promise the best, other localities widely different should also be examined. Chief among these perhaps are the islands of the Pacific.

It is desirable, therefore, to send out observers somewhat as follows :

1. To the desert of Gobi.
2. To the veldt of the Transvaal.
3. To the Samoan Islands.

The observations made at these points could then be repeated elsewhere till the Earth's surface should be known from an astronomic point of view.

Each observer is to be armed with a 6-inch glass, all the glasses made by the same maker (for instance, Alvan Clark & Sons' Corporation), and to report according to the proposed standard scale of "seeing."

It is thus important that the said scale should be agreed to by astronomers generally before the various expeditions start.

Ephemeris for Physical Observations of the Moon for 1903.
By A. C. D. Crommelin.

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
Jan. 1	308·03	+1·48	−4·01	−6·04	7·25	146·4	342·88
2	320·20	+1·49	−5·11	−5·37	7·42	136·4	339·52
3	332·38	+1·50	−6·08	−4·46	7·54	126·3	337·04
4	344·55	+1·50	−6·88	−3·32	7·63	115·8	335·54
5	356·72	+1·51	−7·44	−1·99	7·70	105·0	335·11
6	8·88	+1·52	−7·68	−0·53	7·70	93·9	335·89
7	21·03	+1·52	−7·53	+1·00	7·60	82·4	337·94
8	33·18	+1·53	−6·93	+2·52	7·37	70·0	341·35
9	45·31	+1·53	−5·86	+3·93	7·05	56·2	346·24
10	57·44	+1·53	−4·31	+5·12	6·69	40·1	351·90
11	69·57	+1·53	−2·38	+5·99	6·45	21·7	358·48
12	81·69	+1·54	−0·22	+6·44	6·44	2·0	5·24
13	93·81	+1·53	+1·98	+6·43	6·73	342·9	11·56
14	105·93	+1·53	+4·01	+5·97	7·19	326·1	16·92
15	118·06	+1·53	+5·70	+5·10	7·65	311·8	20·98
16	130·19	+1·52	+6·91	+3·93	7·95	299·6	23·61
17	142·32	+1·52	+7·60	+2·56	8·02	288·6	24·81
18	154·47	+1·51	+7·78	+1·09	7·86	278·0	24·64
19	166·62	+1·50	+7·49	−0·39	7·50	267·0	23·27
20	178·78	+1·50	+6·83	−1·81	7·06	255·2	20·80
21	190·94	+1·49	+5·89	−3·11	6·66	242·2	17·40
22	203·11	+1·48	+4·74	−4·25	6·37	228·1	13·23
23	215·29	+1·47	+3·47	−5·20	6·25	213·7	8·47
24	227·47	+1·46	+2·14	−5·91	6·29	199·9	3·35
25	239·65	+1·46	+0·83	−6·37	6·42	187·4	358·08
26	251·84	+1·45	−0·45	−6·55	6·56	176·1	352·93
27	264·03	+1·44	−1·65	−6·45	6·66	165·7	348·09

Greenwich Midnight.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Geocentric Libration Combined Amount.	Geocentric Libration Direction.	C.
1903.	°	°	°	°	°	°	°
Jan. 28	276·23	+1·43	−2·78	−6·07	6·68	155·4	343·81
29	288·42	+1·43	−3·82	−5·41	6·61	144·8	340·23
30	300·61	+1·42	−4·75	−4·49	6·54	133·4	337·51
31	312·79	+1·41	−5·55	−3·35	6·48	121·1	335·77
Feb. 1	324·98	+1·41	−6·20	−2·04	6·53	108·2	335·09
2	337·16	+1·40	−6·65	−0·60	6·68	95·2	335·59
3	349·33	+1·39	−6·86	+0·90	6·92	82·5	337·29
4	1·50	+1·38	−6·75	+2·40	7·16	70·4	340·24
5	13·66	+1·37	−6·28	+3·78	7·33	59·0	344·45
6	25·81	+1·36	−5·43	+4·97	7·37	47·5	349·74
7	37·96	+1·35	−4·17	+5·89	7·22	35·3	355·86
8	50·10	+1·33	−2·59	+6·44	6·94	21·9	2·40
9	62·23	+1·32	−0·75	+6·56	6·60	6·5	8·82
10	74·36	+1·30	+1·17	+6·23	6·34	349·4	14·60
11	86·49	+1·29	+3·01	+5·47	6·24	331·2	19·29
12	98·63	+1·27	+4·60	+4·35	6·33	313·4	22·62
13	110·76	+1·25	+5·81	+2·97	6·53	297·1	24·48
14	122·89	+1·23	+6·57	+1·45	6·73	282·4	24·87
15	135·04	+1·21	+6·86	−0·11	6·86	269·1	23·92
16	147·19	+1·19	+6·70	−1·61	6·89	256·5	21·75
17	159·35	+1·17	+6·15	−3·00	6·84	244·0	18·57
18	171·51	+1·15	+5·29	−4·21	6·76	231·5	14·53
19	183·68	+1·12	+4·20	−5·20	6·68	218·9	9·87
20	195·85	+1·10	+2·97	−5·96	6·66	206·5	4·81
21	208·03	+1·08	+1·67	−6·46	6·67	194·5	359·55
22	220·22	+1·06	+0·37	−6·68	6·69	183·2	354·35
23	232·41	+1·04	−0·87	−6·61	6·67	172·5	349·40
24	244·61	+1·02	−2·01	−6·26	6·58	162·2	344·93
25	256·81	+1·01	−3·02	−5·62	6·38	151·7	341·12
26	269·01	+0·99	−3·88	−4·71	6·10	140·5	338·13
27	281·22	+0·97	−4·58	−3·57	5·81	127·9	336·09
28	293·42	+0·95	−5·11	−2·23	5·58	113·6	335·14
Mar. 1	305·62	+0·93	−5·46	−0·76	5·51	97·9	335·37
2	317·82	+0·91	−5·60	+0·67	5·64	83·2	336·81
3	330·02	+0·90	−5·55	+2·29	6·00	67·6	339·50
4	342·20	+0·88	−5·25	+3·70	6·43	54·8	343·40
5	354·39	+0·86	−4·71	+4·93	6·82	43·7	348·37
6	6·56	+0·83	−3·90	+5·88	7·06	33·6	354·18

Greenwich Midnight.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
Mar. 7	18·73	+0·81	−2·85	+6·50	7·10	23·7	0·45
8	30·89	+0·79	−1·59	+6·71	6·89	13·3	6·73
9	43·05	+0·76	−0·19	+6·50	6·50	1·7	12·60
10	55·20	+0·74	+1·26	+5·86	5·99	347·9	17·60
11	67·35	+0·71	+2·65	+4·85	5·52	331·4	21·44
12	79·50	+0·68	+3·86	+3·54	5·24	312·5	23·88
13	91·64	+0·65	+4·80	+2·02	5·21	292·8	24·89
14	103·79	+0·62	+5·40	+0·42	5·42	274·4	24·48
15	115·94	+0·59	+5·63	−1·17	5·75	258·3	22·77
16	128·10	+0·56	+5·48	−2·65	6·09	244·2	19·90
17	140·26	+0·53	+5·00	−3·97	6·38	231·6	16·07
18	152·43	+0·51	+4·22	−5·06	6·59	219·8	11·53
19	164·60	+0·48	+3·21	−5·90	6·71	208·5	6·50
20	176·78	+0·45	+2·04	−6·48	6·80	197·5	1·23
21	188·97	+0·42	+0·78	−6·78	6·82	186·6	355·96
22	201·16	+0·39	−0·49	−6·78	6·80	175·9	350·92
23	213·36	+0·37	−1·69	−6·49	6·71	165·4	346·29
24	225·57	+0·34	−2·79	−5·92	6·54	154·8	342·26
25	237·78	+0·32	−3·71	−5·07	6·28	143·8	338·99
26	249·99	+0·29	−4·39	−4·02	5·96	132·5	336·62
27	262·21	+0·27	−4·91	−2·65	5·58	118·4	335·31
28	274·43	+0·25	−5·14	−1·17	5·27	102·8	335·16
29	286·65	+0·22	−5·11	+0·40	5·13	85·5	336·28
30	298·87	+0·20	−4·85	+1·98	5·24	67·8	338·68
31	311·08	+0·18	−4·36	+3·46	5·56	51·6	342·36
Apr. 1	323·30	+0·15	−3·67	+4·76	6·01	37·6	347·17
2	335·50	+0·13	−2·82	+5·79	6·44	26·0	352·86
3	347·70	+0·10	−1·85	+6·48	6·74	16·0	359·04
4	359·90	+0·07	−0·79	+6·77	6·82	6·7	5·31
5	12·08	+0·05	+0·30	+6·65	6·66	357·4	11·20
6	24·27	+0·02	+1·37	+6·12	6·27	347·4	16·33
7	36·44	−0·01	+2·38	+5·22	5·74	335·5	20·41
8	48·61	−0·04	+3·26	+4·01	5·17	320·9	23·23
9	60·78	−0·07	+3·97	+2·56	4·72	302·8	24·71
10	72·94	−0·11	+4·47	+1·00	4·58	282·6	24·81
11	85·11	−0·14	+4·72	−0·60	4·76	262·8	23·57
12	97·27	−0·17	+4·71	−2·14	5·17	245·6	21·12
13	109·44	−0·20	+4·42	−3·53	5·66	231·4	17·61

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
Apr. 14	121·61	−0·23	+3·88	−4·72	6·11	219·4	13·26
15	133·79	−0·26	+3·09	−5·67	6·46	208·6	8·32
16	145·97	−0·29	+2·11	−6·34	6·68	198·4	3·05
17	158·16	−0·32	+0·97	−6·73	6·80	188·2	357·72
18	170·35	−0·34	−0·27	−6·82	6·83	177·7	352·54
19	182·55	−0·37	−1·53	−6·62	6·80	167·0	347·76
20	194·76	−0·39	−2·74	−6·13	6·72	155·9	343·53
21	206 97	−0·42	−3·83	−5·37	6·60	144·5	340·01
22	219·18	−0·44	−4·73	−4·36	6·44	132·7	337·34
23	231·41	−0·47	−5·38	−3·12	6·22	120·1	335·65
24	243·64	−0·49	−5·74	−1·69	5·98	106·4	335·06
25	255·87	−0·51	−5·73	−0·14	5·73	91·4	335·72
26	268·11	−0·53	−5·39	+1·46	5·59	74·8	337·69
27	280·34	−0·56	−4·72	+3·00	5·59	57·6	341·01
28	292·58	−0·58	−3·75	+4·39	5·77	40·5	345·59
29	304·81	−0·60	−2·58	+5·52	6·10	25·1	351·20
30	317·05	−0·62	−1·28	+6·31	6·44	11·5	357·46
May 1	329·27	−0·64	+0·05	+6·69	6·69	359·6	3·87
2	341·49	−0·67	+1·31	+6·65	6·78	348·9	9·95
3	353·70	−0·69	+2·43	+6·20	6·66	338·6	15·30
4	5·90	−0·72	+3·36	+5·38	6·34	328·0	19·60
5	18·10	−0·75	+4·09	+4·24	5·90	316·0	22·69
6	30·29	−0·77	+4·58	+2·88	5·41	302·2	24·48
7	42·48	−0·80	+4·85	+1·38	5·04	285·9	24·92
8	54·66	−0·82	+4·92	−0·18	4·92	267·9	24·08
9	66·85	−0·85	+4·77	−1·70	5·06	250·4	22·02
10	79·03	−0·87	+4·43	−3·11	5·42	234·9	18·86
11	91·21	−0·90	+3·90	−4·34	5·83	221·9	14·78
12	103·39	−0·92	+3·18	−5·35	6·22	210·7	10·00
13	115·57	−0·95	+2·30	−6·09	6·51	200·7	4·80
14	127·76	−0·97	+1·25	−6·55	6·67	190·8	359·42
15	139·95	−0·99	+0·09	−6·72	6·72	180·8	354·16
16	152·15	−1·01	−1·15	−6·59	6·69	170·1	349·22
17	164·35	−1·03	−2·43	−6·18	6·64	158·5	344·81
18	176·56	−1·05	−3·67	−5·51	6·62	146·3	341·07
19	188·78	−1·07	−4·80	−4·58	6·64	133·7	338·14
20	201·00	−1·09	−5·75	−3·44	6·70	120·9	336·12
21	213·23	−1·10	−6·42	−2·11	6·76	108·2	335·14

Greenwich Midnight.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
May 22	225·46	−1·12	−6·73	−0·64	6·76	95·4	335·31
23	237·71	−1·13	−6·65	+0·91	6·71	82·2	336·74
24	249·94	−1·15	−6·13	+2·46	6·61	68·1	339·52
25	262·19	−1·16	−5·17	+3·89	6·47	53·0	343·66
26	274·44	−1·18	−3·83	+5·11	6·39	36·9	348·99
27	286·69	−1·19	−2·22	+6·01	6·41	20·3	355·20
28	298·93	−1·20	−0·47	+6·51	6·53	4·1	1·81
29	311·18	−1·22	+1·27	+6·57	6·69	349·1	8·25
30	323·41	−1·24	+2·85	+6·19	6·81	335·3	14·00
31	335·64	−1·25	+4·16	+5·42	6·83	322·5	18·70
June 1	347·87	−1·27	+5·14	+4·33	6·72	310·1	22·12
2	0·08	−1·29	+5·77	+3·01	6·51	297·6	24·21
3	12·30	−1·30	+6·06	+1·54	6·25	284·3	24·94
4	24·50	−1·32	+6·06	+0·02	6·06	270·2	24·38
5	36·70	−1·34	+5·81	−1·47	5·99	255·8	22·62
6	48·90	−1·35	+5·35	−2·86	6·07	241·9	19·76
7	61·09	−1·37	+4·71	−4·09	6·24	229·0	15·95
8	73·29	−1·38	+3·92	−5·11	6·44	217·5	11·38
9	85·48	−1·40	+3·01	−5·88	6·61	207·1	6·30
10	97·67	−1·41	+2·04	−6·37	6·69	197·8	0·96
11	109·86	−1·42	+0·84	−6·58	6·63	187·3	355·63
12	122·06	−1·43	−0·38	−6·50	6·51	176·7	350·58
13	134·26	−1·44	−1·67	−6·14	6·36	164·8	346·00
14	146·46	−1·45	−2·97	−5·52	6·27	151·7	342·06
15	158·67	−1·45	−4·24	−4·65	6·30	137·6	338·91
16	170·88	−1·46	−5·41	−3·58	6·49	123·5	336·61
17	183·10	−1·46	−6·40	−2·32	6·81	109·9	335·32
18	195·32	−1·47	−7·12	−0·93	7·18	97·4	335·10
19	207·56	−1·47	−7·49	+0·55	7·51	85·8	336·05
20	219·79	−1·48	−7·44	+2·04	7·71	74·7	338·26
21	232·04	−1·48	−6·89	+3·46	7·71	63·3	341·79
22	244·28	−1·48	−5·85	+4·72	7·52	51·1	346·60
23	256·53	−1·49	−4·35	+5·71	7·18	37·3	352·49
24	268·79	−1·49	−2·49	+6·33	6·80	21·5	359·06
25	281·04	−1·49	−0·43	+6·51	6·53	3·8	5·77
26	293·30	−1·50	+1·63	+6·23	6·44	345·3	12·01
27	305·55	−1·50	+3·52	+5·52	6·54	327·5	17·28
28	317·79	−1·51	+5·07	+4·45	6·75	311·3	21·25

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
June 29	330·03	−1·51	+6·21	+3·12	6·95	296·7	23·78
30	342·26	−1·52	+6·92	+1·64	7·11	283·3	24·89
July 1	354·49	−1·52	+7·19	+0·11	7·19	270·9	24·62
2	6·70	−1·53	+7·10	−1·38	7·23	259·0	23·11
3	18·92	−1·53	+6·70	−2·77	7·25	247·5	20·48
4	31·13	−1·53	+6·05	−4·00	7·26	236·5	16·88
5	43·33	−1·53	+5·21	−5·02	7·23	226·1	12·50
6	55·53	−1·53	+4·24	−5·80	7·17	216·2	7·56
7	67·73	−1·53	+3·15	−6·31	7·05	206·5	2·29
8	79·92	−1·53	+1·99	−6·53	6·83	197·0	356·96
9	92·11	−1·53	+0·76	−6·47	6·51	186·7	351·82
10	104·31	−1·53	−0·51	−6·13	6·15	175·2	347·09
11	116·50	−1·52	−1·81	−5·53	5·82	161·9	342·97
12	128·70	−1·52	−3·10	−4·68	5·61	146·5	339·60
13	140·91	−1·51	−4·35	−3·63	5·66	129·8	337·11
14	153·11	−1·50	−5·50	−2·40	6·00	113·6	335·56
15	165·32	−1·49	−6·49	−1·05	6·57	99·2	335·04
16	177·54	−1·48	−7·24	+0·38	7·25	87·0	335·64
17	189·76	−1·48	−7·66	+1·83	7·88	76·6	337·41
18	201·98	−1·47	−7·67	+3·22	8·32	67·2	340·40
19	214·22	−1·46	−7·20	+4·48	8·48	58·1	344·62
20	226·46	−1·45	−6·24	+5·51	8·32	48·6	349·97
21	238·70	−1·44	−4·79	+6·22	7·85	37·6	356·19
22	250·95	−1·43	−2·94	+6·53	7·16	24·2	2·86
23	263·21	−1·42	−0·94	+6·38	6·45	8·4	9·43
24	275·46	−1·41	+1·32	+5·77	5·92	347·1	15·19
25	287·71	−1·41	+3·36	+4·76	5·82	324·8	19·83
26	299·96	−1·40	+5·09	+3·43	6·14	304·0	23·02
27	312·20	−1·39	+6·40	+1·91	6·68	286·6	24·67
28	324·44	−1·38	+7·24	+0·32	7·25	272·5	24·84
29	336·67	−1·38	+7·62	−1·25	7·72	260·7	23·65
30	348·90	−1·37	+7·58	−2·70	8·05	250·4	21·25
31	1·12	−1·36	+7·17	−3·97	8·20	241·0	17·82
Aug. 1	13·33	−1·35	+6·47	−5·03	8·20	232·1	13·59
2	25·54	−1·34	+5·57	−5·83	8·06	223·7	8·75
3	37·74	−1·33	+4·48	−6·36	7·78	215·2	3·55
4	49·94	−1·31	+3·28	−6·61	7·38	206·4	358·22
5	62·14	−1·30	+2·06	−6·57	6·89	197·4	353·03

Greenwich Midnight.	Selenographical Colong.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
Aug. 6	74·33	−1·28	+0·78	−6·24	6·29	187·1	348·18
7	86·52	−1·26	−0·50	−5·65	5·67	174·9	343·89
8	98·70	−1·25	−1·77	−4·81	5·13	159·8	340·33
9	110·89	−1·23	−2·99	−3·75	4·80	141·4	337·60
10	123·09	−1·21	−4·15	−2·52	4·86	121·3	335·82
11	135·28	−1·19	−5·19	−1·16	5·32	102·6	335·06
12	147·48	−1·17	−6·07	+0·27	6·08	87·5	335·39
13	159·68	−1·15	−6·73	+1·72	6·95	75·7	336·85
14	171·88	−1·13	−7·10	+3·11	7·76	66·3	339·48
15	184·09	−1·11	−7·11	+4·37	8·35	58·4	343·25
16	196·31	−1·09	−6·73	+5·43	8·65	51·1	348·12
17	208·53	−1·07	−5·91	+6·20	8·56	43·6	353·90
18	220·76	−1·05	−4·66	+6·61	8·09	35·2	0·25
19	233·00	−1·03	−3·03	+6·59	7·25	24·7	6·72
20	245·24	−1·01	−1·15	+6·13	6·24	10·6	12·80
21	257·49	−0·99	+0·84	+5·23	5·30	350·9	17·96
22	269·73	−0·97	+2·77	+3·97	4·84	325·1	21·82
23	281·98	−0·95	+4·48	+2·44	5·10	298·6	24·17
24	294·22	−0·94	+5·84	+0·79	5·89	277·7	24·96
25	306·46	−0·92	+6·77	−0·87	6·82	262·7	24·25
26	318·69	−0·90	+7·27	−2·43	7·66	251·5	22·19
27	330·92	−0·88	+7·33	−3·81	8·26	242·5	19·00
28	343·13	−0·86	+7·01	−4·96	8·59	234·7	14·89
29	355·34	−0·84	+6·36	−5·84	8·63	227·5	10·12
30	7·55	−0·82	+5·45	−6·43	8·43	220·3	4·95
31	19·75	−0·80	+4·36	−6·72	8·01	213·0	359·61
Sept. 1	31·95	−0·77	+3·14	−6·73	7·43	205·0	354·35
2	44·14	−0·75	+1·87	−6·44	6·71	196·2	349·39
3	56·33	−0·73	+0·58	−5·88	5·91	185·6	344·94
4	68·51	−0·70	−0·68	−5·06	5·11	172·3	341·16
5	80·69	−0·67	−1·88	−4·02	4·44	154·9	338·20
6	92·87	−0·64	−2·97	−2·78	4·07	133·1	336·16
7	105·04	−0·61	−3·94	−1·41	4·18	109·7	335·15
8	117·22	−0·59	−4·76	+0·05	4·76	89·4	335·22
9	129·40	−0·56	−5·40	+1·53	5·61	74·2	336·42
10	141·58	−0·53	−5·82	+2·95	6·53	63·1	338·79
11	153·77	−0·50	−5·99	+4·25	7·35	54·6	342·30
12	165·96	−0·47	−5·87	+5·35	7·95	47·6	346·87

Greenwich Midnight.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Combined Amount.	Direction.	C.
1903	°	°	°	°	°	°	°
Sept. 13	178·16	−0·45	−5·44	+6·18	8·24	41·4	352·32
14	190·36	−0·42	−4·70	+6·66	8·15	35·2	358·37
15	202·57	−0·39	−3·66	+6·75	7·68	28·5	4·66
16	214·78	−0·37	−2·34	+6·42	6·83	20·0	10·73
17	227·00	−0·34	−0·84	+5·67	5·73	8·4	16·10
18	239·23	−0·32	+0·77	+4·53	4·59	350·4	20·42
19	251·46	−0·29	+2·35	+3·10	3·89	322·8	23·36
20	263·69	−0·27	+3·89	+1·46	4·16	290·6	24·81
21	275 92	−0·25	+4·98	−0·24	4·99	267·2	24·73
22	288·15	−0·23	+5·86	−1·90	6·16	252·0	23·19
23	300·38	−0·20	+6·36	−3·40	7·21	241·9	20·37
24	312·60	−0·18	+6·49	−4·67	7·99	234·3	16 49
25	324·81	−0·15	+6·24	−5·67	8·44	227·7	11·82
26	337·02	−0·13	+5·66	−6·36	8·52	221·7	6·65
27	349·22	−0·10	+4·81	−6·75	8·29	215·5	1·26
28	1·42	−0·07	+3·74	−6·83	7·79	208·7	355·90
29	13·61	−0·05	+2·53	−6·61	7·08	200 9	350·81
30	25·79	−0·02	+1·24	−6·11	6·23	191·5	346·19
Oct. 1	37·97	+0·01	−0·04	−5·35	5·35	179·6	342·20
2	50·15	+0·04	−1·27	−4·36	4·54	163·8	338·99
3	62·31	+0·07	−2·39	−3 16	3·96	142·9	336·66
4	74·48	+0·10	−3·34	−1·80	3·79	118·3	335·33
5	86·64	+0·13	−4·09	−0·34	4·10	94·8	335·08
6	98·80	+0·17	−4·62	+1·17	4·77	75·8	335 98
7	110·96	+0·20	−4·90	+2·64	5·57	61·7	338·08
8	123·12	+0·23	−4·93	+4·00	6·35	50·9	341·37
9	135·29	+0·26	−4·73	+5·16	7·00	42·5	345·75
10	147·45	+0·29	−4·29	+6·05	7·41	35·3	351·06
11	159·62	+0·32	−3·65	+6 60	7·54	28·9	357·00
12	171·80	+0·34	−2·82	+6 77	7·33	22·6	3·20
13	183·98	+0·37	−1·86	+6·54	6·80	15·9	9·24
14	196 18	+0·40	−0·79	+5·91	5·96	7·6	14·70
15	208·37	+0·42	+0·35	+4·90	4·91	355·9	19·23
16	220·58	+0·44	+1·50	+3·59	3·89	337·3	22·54
17	232·78	+0·47	+2·60	+2·05	3·31	308·2	24·47
18	245·00	+0·49	+3·60	+0·39	3·62	276·2	24·94
19	257·21	+0·51	+4·43	−1·27	4·61	254·0	23·98
20	269·43	+0·53	+5·04	−2·83	5·78	240·7	21·65

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1903	°	°	°	°	°	°	°
Oct. 21	281·64	+0·56	+5·39	−4·20	6·83	232·1	18·14
22	293·85	+0·58	+5·44	−5·30	7·60	225·7	13·70
23	306·05	+0·60	+5·18	−6·11	8·01	220·3	8·61
24	318·26	+0·63	+4·62	−6·61	8·06	214·9	3·18
25	330·45	+0·65	+3·79	−6·78	7·77	209·2	357·70
26	342·64	+0·68	+2·74	−6·65	7·19	202·4	352·45
27	354·83	+0·70	+1·52	−6·23	6·41	193·7	347·63
28	7·01	+0·73	+0·23	−5·55	5·56	182·4	343·42
29	19·18	+0·75	−1·09	−4·63	4·75	166·8	339·95
30	31·35	+0·78	−2·31	−3·51	4·20	146·7	337·33
31	43·51	+0·81	−3·39	−2·21	4·05	123·1	335·66
Nov. 1	55·66	+0·83	−4·25	−0·79	4·32	100·5	335·03
2	67·81	+0·86	−4·82	+0·70	4·87	81·7	335·54
3	79·96	+0·89	−5·07	+2·18	5·52	66·7	337·26
4	92·10	+0·91	−4·99	+3·58	6·15	54·3	340·20
5	104·24	+0·94	−4·59	+4·81	6·65	43·7	344·34
6	116·38	+0·96	−3·89	+5·78	6·96	33·9	349·52
7	128·52	+0·99	−2·99	+6·42	7·08	25·0	355·47
8	140·67	+1·01	−1·94	+6·66	6·94	16·2	1·77
9	152·82	+1·03	−0·84	+6·50	6·55	7·4	7·96
10	164·99	+1·05	+0·26	+5·94	5·95	357·5	13·60
11	177·15	+1·07	+1·28	+5·01	5·17	345·7	18·32
12	189·32	+1·09	+2·21	+3·78	4·38	329·7	21·87
13	201·50	+1·11	+3·01	+2·34	3·81	307·9	24·11
14	213·68	+1·12	+3·68	+0·76	3·76	281·7	24·96
15	225·88	+1·14	+4·21	−0·84	4·29	258·7	24·43
16	238·07	+1·15	+4·60	−2·38	5·18	242·6	22·56
17	250·27	+1·17	+4·82	−3·76	6·12	232·0	19·49
18	262·47	+1·18	+4·86	−4·92	6·91	224·6	15·39
19	274·67	+1·20	+4·69	−5·80	7·46	219·0	10·51
20	286·87	+1·21	+4·28	−6·37	7·67	213·9	5·14
21	299·06	+1·23	+3·63	−6·62	7·55	208·7	359·62
22	311·25	+1·24	+2·75	−6·57	7·12	202·7	354·20
23	323·44	+1·26	+1·67	−6·22	6·44	195·0	349·17
24	335·62	+1·28	+0·43	−5·60	5·62	184·4	344·72
25	347·80	+1·29	−0·91	−4·75	4·84	169·2	341·00
26	359·97	+1·31	−2·26	−3·69	4·33	148·5	338·11
27	12·12	+1·32	−3·54	−2·47	4·32	124·9	336·12

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.	C.
1903.	°	°	°	°	°	°	°
Nov. 28	24·28	+1·34	−4·65	−1·12	4·78	103·5	335·13
29	36·43	+1·36	−5·51	+0·30	5·52	86·9	335·21
30	48·58	+1·37	−6·02	+1·75	6·27	73·8	336·44
Dec. 1	60·71	+1·39	−6·04	+3·15	6·81	62·5	338·89
2	72·85	+1·40	−5·80	+4·41	7·28	52·8	342·58
3	84·98	+1·42	−5·05	+5·45	7·43	42·8	347·44
4	97·10	+1·43	−3·92	+6·17	7·31	32·4	353·27
5	109·23	+1·44	−2·53	+6·52	6·99	21·2	359·66
6	121·36	+1·45	−1·00	+6·43	6·51	8·8	6·14
7	133·49	+1·46	+0·55	+5·92	5·95	354·7	12·16
8	145·63	+1·47	+1·96	+5·03	5·40	338·7	17·29
9	157·77	+1·47	+3·17	+3·83	4 97	320·4	21·20
10	169·92	+1·48	+4·12	+2·41	4·77	300·3	23·75
11	182 08	+1·48	+4·80	+0·86	4·88	280·2	24·91
12	194·25	+1·48	+5·23	−0 70	5·28	262·4	24·67
13	206·42	+1·49	+5·45	−2·20	5·88	248·0	23·13
14	218·60	+1·49	+5·46	−3·56	6·52	236·9	20·40
15	230·78	+1·49	+5·31	−4·71	7·10	228·4	16·63
16	242·97	+1·49	+4·99	−5·61	7·50	221·6	12·04
17	255·16	+1·50	+4·51	−6·21	7·68	216·0	6·83
18	267 35	+1·50	+3·86	−6·51	7·57	210·7	1·35
19	279·54	+1·50	+3·03	−6·49	7·16	205·0	355·87
20	291·73	+1·50	+2·03	−6·18	6 51	198·2	350·69
21	303·92	+1·51	+0·86	−5·60	5·66	188·7	346·02
22	316·10	+1·51	−0·44	−4·78	4 80	174·7	342·05
23	328·28	+1·51	−1·82	−3·76	4·18	154·2	338·90
24	340·45	+1·52	−3·22	−2·58	4·13	128·7	336·64
25	352·61	+1·52	−4·55	−1·28	4·73	105·7	335·34
26	4·77	+1·52	−5·72	+0·10	5·72	89·0	335 05
27	16 92	+1·52	−6·64	+1·50	6·81	77·3	335·85
28	29·07	+1·52	−7·21	+2·86	7·75	68·4	337·79
29	41·22	+1·52	−7·35	+4·12	8·43	60·7	340·91
30	53·35	+1·52	−6·99	+5·20	8·71	53·4	345·22
31	65·48	+1·52	−6·12	+6·00	8·57	45·6	350·61

The longitudes are reckoned in the plane of the Moon's equator, the axis of reference being the radius which passes through the mean centre of the visible disc. This axis therefore rotates with the Moon, and is not fixed in space.

The inclination of the Moon's equator to the ecliptic is taken as 1°·523, the value used in the *Connaissance des Temps*, that given by the *Nautical Almanac* being 1°·536.

The physical librations in longitude and latitude, as given by Professor Franz's formulæ, have been applied; their values are taken from the *Berliner Jahrbuch* for the days given there, and interpolated by a graphical method for the other days. But the signs in the *Jahrbuch* require to be reversed in order to reduce to the system used here.

The colongitude of the Sun is 90° (or 450°) *minus* his selenographical longitude. It also is the selenographical longitude of the morning terminator reckoned eastward from the mean centre of the disc. Hence its value is approximately 270°, 0°, 90°, 180° at new Moon, first quarter, full Moon, last quarter respectively. The longitude of the evening terminator is of course 180° greater or less than that of the morning one.

When the geocentric libration in longitude is positive, the region brought into view is on the west limb; when negative, on the east.

When the geocentric libration in latitude is positive, the region brought into view is at the Moon's north pole; when negative, at the south.

The column "Combined Amount" gives the distance between the apparent and mean centres of the disc, and the column "Direction" gives the position-angle of the apparent centre from the mean centre, or, which is the same thing, the position-angle of the region which is most carried into view by libration. The angles are reckoned eastward from the northern extremity of the Moon's axis.

C denotes the geocentric position-angle of the northern extremity of the Moon's axis measured eastward from the northernmost point of the disc. It has been computed by the second formula given in the Preface to the *Nautical Almanac*, but the co-ordinates of the Moon's equator have been taken from the *Connaissance des Temps*, so as to make this column consistent with the rest of the ephemeris.

The terms "East" and "West" are used throughout with reference to our sky, and not as they would appear to an observer on the Moon.

I give the method for finding the altitude of the Sun at a given point on the Moon whose position is defined: (1) by selenographical longitude and latitude; (2) by direction cosines.

In either case the Sun's selenographical colongitude and latitude (K, L supposed) must be found by interpolation from the ephemeris for the given time.

In the first case let the given point be in the position longitude M, latitude N. Longitudes are reckoned from the meridian passing through the mean centre of the disc, and the positive direction is that towards Mare Crisium. North latitudes are considered positive.

Then

$$\text{sine Sun's altitude} = \sin L \sin N + \cos L \cos N \sin (K+M).$$

In the second case let ξ, η, ζ be the direction cosines of the given point. The axes are (1) that diameter of the Moon's equator which is 90° from the mean centre of the disc; (2) the Moon's polar axis; (3) the diameter through the mean centre of the disc. The positive directions are as above. Mr. Saunder has issued some maps of portions of the Moon's surface from which the co-ordinates ξ, η, ζ can be taken at sight.

Then the Sun's direction cosines are:

$$\cos K \cos L, \sin L, \sin K \cos L,$$

and sine Sun's altitude

$$= \xi \cos K \cos L + \eta \sin L + \zeta \sin K \cos L.$$

Neither formula is convenient when the Sun's altitude is very great, for an angle near 90° cannot be accurately determined from its sine. However, when the Sun is high the shadows are so inconspicuous that it is not necessary to compute his altitude with great accuracy.

Benvenue, 55 *Ulundi Road, Blackheath, S.E.:*
1902 *September* 1.

Erratum in Mr. Aldis's paper, vol. lxii. p. 636, line 5:

for $a \times \cdot 20878$ *read* $a \times \cdot 41745$.